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COMPUTER PROGRAMS FOR THE DESIGN OF ACTIVE RC FILTERS

by

L.J. Richards

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17\. Abstract

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SUMMARY

This Report describes three Algol 68-R programs which may be used as aids to the design of active RC filters. Two programs calculate the performance of standard all-pole filter types, the first program calculating the attenuation and the second the group delay. The third calculates component values needed to implement the filter as a cascade connection of first or second order sections. Sensitivity and gain performance may also be calculated. Algorithms are described which ensure that where possible component values lie within specified bounds and take standard values. Specifications of the programs are given, together with examples of their use.

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LIST OF CONTENTS

038

1 INTRODUCTION

The process of designing an analogue filter can be split into the following steps:

(i) Decide on the mathematical form of the filter, *ie* compute its transfer function.

(ii) Decide on a particular type of circuit to realise the transfer function.

(iii) Compute the component values for the circuit.

(iv) Investigate the performance of the circuit thus designed. If not satisfactory repeat one or more of the previous steps.

None of these steps can be made completely automatic but the use of computer programs can ease the designer's task. This Report describes three such programs written for use on the ICL 1906S computer.

Two programs calculate attenuation and delay performance of low-pass, high-pass and band-pass filters of standard all-pole types including Butterworth, Chebyshev and Bessel types. The first program, ACFA, calculates the attenuation performance and the second, ACFD, calculates the delay performance. The third program, ACFC, allows the user to specify one of a number of standard active RC circuit configurations and calculates the values of the resistance and capacitance values required. For some circuit types this program also evaluates the sensitivity of each passive component, the input and output impedances of the circuit and the filter performance with non-ideal active elements.

We now discuss in more detail the filter types and circuit configurations chosen. The transfer functions considered are of the all-pole type, that is their transfer functions can be written in the form

$$H(s) = \frac{K s^I}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

where $K, a_0, a_1, \ldots, a_{n-1}$ are real constants and $I = 0$ or n or $n/2$ for low-pass or high-pass or band-pass filters, respectively. Bandstop and all-pass filters are not considered. This form of transfer function encompasses for example Butterworth and Chebyshev filters but does not include filters such as Elliptic or Inverse Chebyshev which have points of infinite attenuation at finite frequencies.

The filter types considered are listed and defined in section 3 below. The common practice is followed of defining the transfer function in normalised form. In section 2 therefore we describe the normalisation and denormalisation procedures and discuss the frequency transformations used to obtain, for example, a high-pass filter transfer function from a low-pass prototype. Also in section 2 we define some of the filter parameters used in the rest of the Report.

The circuits considered are active RC circuits using operational amplifiers as the active elements. Each circuit consists of a number of subcircuits or sections connected in cascade. Each subcircuit realises a first or second order transfer function. Only one type of first order section is considered, but there is a choice of six configurations for second order sections. These types are listed and compared in section 4. Although different circuits may use different section types it is assumed that any one circuit is composed of only one section type.

In section 5 we discuss the choice of component values and explain how the computer program uses preferred capacitor values and ensures that both capacitor and resistor values are within specified bounds. The calculation of differential sensitivities and large change sensitivities is explained in section 6 and in section 7 we discuss the calculation of input and output impedances, voltage range and gain performance with non-ideal operational amplifiers. The next four sections (8, 9, 10 and 11) give detailed specifications of the three computer programs and give examples of their use.

Active filters provide an alternative to traditional passive filters for frequencies up to a few hundred kilohertz. For frequencies up to a few kilohertz the active filter becomes particularly attractive since it is more amenable to miniaturisation than its passive counterpart. Active filters have been studied by May and Brown[11], who designed a low frequency narrow-band filter for airborne use. Kimbell[12] used active circuits to realise pulse forming filters. Norman[13] considered the application of active transitional Butterworth-Thomson filters as pre-sampling filters in telemetry systems. It is anticipated that active filters will continue to be of interest to RAE for low-frequency applications, for example in the processing of voice and data signals, and it is to meet this requirement that the present programme of work has been undertaken.

2 NORMALISATION AND DENORMALISATION OF TRANSFER FUNCTIONS

The transfer function is defined to be the ratio of the Laplace Transform of the output to that of the input and we denote it by $H(s)$. It is convenient

to write an nth order transfer function as the product of a number of first or second order transfer functions

$$H(s) = \prod_{k=1}^{n/2} \frac{C_k s^2 + D_k s + E_k}{s^2 + A_k s + B_k}$$

if n is even, or

$$H(s) = \frac{ds + e}{s + a} \prod_{k=1}^{(n-1)/2} \frac{C_k s^2 + D_k s + E_k}{s^2 + A_k s + B_k}$$

if n is odd.

For low-pass filters $C_k = D_k = d = 0 \quad (k = 1, 2, \ldots)$

For high-pass filters $D_k = E_k = e = 0$.

For band-pass filters $C_k = E_k = d = e = 0$ (only even orders are allowed).

The standard filter types of section 3 are defined only for low-pass 'prototype' filters with cut-off frequency 1 rad/s and unity gain. We can proceed from the transfer function of such a normalised filter to obtain the transfer function of a low-pass, high-pass or band-pass filter of arbitrary cut-off frequency by the rules given in Appendix A.

The second order transfer function is often written as

$$\frac{G\omega_0^2}{s^2 + \omega_0 s/Q + \omega_0^2} \qquad \text{for a low-pass filter}$$

or

$$\frac{Gs^2}{s^2 + \omega_0 s/Q + \omega_0^2} \qquad \text{for a high-pass filter}$$

or

$$\frac{G\omega_0 s/Q}{s^2 + \omega_0 s/Q + \omega_0^2} \qquad \text{for a band-pass filter,}$$

where G is the gain. The quality factor Q and the resonant frequency ω_0 are related to the A and B coefficients by

$$\omega_0 = \sqrt{B}$$

$$Q = \sqrt{B}/A \ .$$

Some of the section types of section 4 invert the output with respect to the input, so that for these sections G should strictly be negative. In this Report, however, G is assumed to be positive, *ie* G is the absolute value of the gain. Thus, the most general form of the low-pass transfer function, for example, is

$$\pm \frac{G\omega_0^2}{s^2 + \omega_0 s/Q + \omega_0^2} \ .$$

Section 4.8 indicates which of the sections are inverting and which non-inverting.

Finally, the definition of the cut-off frequency must be given. For filters having a monotonic amplitude response in the pass-band (*eg* Butterworth, Papoulis, Halpern, Synchronously Tuned) it is conventional that the cut-off frequency is that frequency at which the amplitude is $1/\sqrt{2}$ times its maximum value, *ie* at which the attenuation is 3 dB. For filters having an oscillatory amplitude response in the pass-band (*eg* Chebyshev, Ultraspherical) the cut-off frequency is taken to be the highest frequency at which the amplitude differs from its maximum value by the amount of the ripple. This is illustrated in Fig 8.

3 FILTER TYPES

In this section we define those filter types used in the programs. The filter types are: Butterworth, Chebyshev, Gaussian, Bessel, Ultraspherical, Papoulis, Halpern and Synchronously Tuned. The first four of these are well-known[1] and details of all the types are given in Ref 2, so here we give the bare definitions only. In each case we give the definition for an nth order low-pass filter having cut-off frequency 1 rad/s and minimum attenuation 0 dB. For typographical convenience we define the inverse of the transfer function, $1/H(s)$, rather than the transfer function $H(s)$ itself. For some filters it is more convenient to quote $|H(s)|^2$ rather than $H(s)$. To obtain the transfer function in these cases one solves $|H(s)|^2 = 0$ and rejects those roots with positive real parts. The remaining roots are those of $H(s) = 0$.

Once one has the transfer function it is straightforward to calculate the gain and attenuation at any frequency, ω. The gain is simply

$$|H(s)|$$

where $s = j\omega$, and the attenuation is

$$-20 \log_{10}|H(s)| \quad .$$

The group delay, $D(\omega)$, defined by

$$D(\omega) \quad = \quad - \frac{d}{d\omega} \arg H(s)$$

is slightly more complicated. It can be found from the formula

$$D(\omega) \quad = \quad \sum_{k=1}^{n/2} \frac{A_k(B_k + \omega^2)}{(B_k - \omega^2)^2 + (A_k\omega)^2}$$

if n is even, or

$$\frac{a}{a^2 + \omega^2} + \sum_{k=1}^{(n-1)/2} \frac{A_k(B_k + \omega^2)}{(B_k - \omega^2)^2 + (A_k\omega)^2}$$

if n is odd.

It should be mentioned here that not all the filter types listed above are available in all three programs. Papoulis, Halpern and Synchronously Tuned filters are not available in either of programs ACFC (for calculation of component values) or ACFD (for calculation of delay). This is not too great a restriction on the usefulness of the programs, since these three filter types are relatively rarely used.

3.1 Butterworth filter

$$1/H(s) \quad = \quad \prod_{k=1}^{n/2} \left[s^2 + 2 \cos\Big((2k - 1)\pi/(2n)\Big) s + 1 \right]$$

if n is even, or

$$(s + 1) \prod_{k=1}^{(n-1)/2} \left[s^2 + 2 \cos(k\pi/n) s + 1 \right]$$

if n is odd.

This gives an attenuation of 3 dB at $\omega = 1$.

3.2 Chebyshev filter

$$1/H(s) = \prod_{k=1}^{n/2} (s - p_k + jq_k)(s - p_k - jq_k)$$

if n is even, or

$$\left(s + \sinh\left(\frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon}\right)\right) \prod_{k=1}^{(n-1)/2} (s - p_k + jq_k)(s - p_k - jq_k)$$

if n is odd,

where
$$p_k = - \sin \frac{\pi}{2} \left(\frac{1 + 2k}{n}\right) \sinh\left(\frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon}\right)$$

$$q_k = \cos \frac{\pi}{2} \left(\frac{1 + 2k}{n}\right) \cosh\left(\frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon}\right)$$

$$\varepsilon^2 = 10^{0.1\gamma} - 1$$

and γ is the attenuation at $\omega = 1$.

3.3 Gaussian filter

$$1/|H(s)|^2 = 1 + p\omega^2 + \frac{p^2\omega^4}{2!} + \frac{p^3\omega^6}{3!} + \ldots + \frac{p^n\omega^{2n}}{n!} ,$$

where p is a constant chosen so that the attenuation at $\omega = 1$ is 3 dB.

3.4 Bessel filter

$$1/H(s) = B_n(D_0 s)/B_n(0)$$

where $B_n(x)$ is the nth order Bessel polynomial which can be defined recursively by

$$B_0(x) = 1$$

$$B_1(x) = x + 1$$

$$B_n(x) = (2n - 1)B_{n-1}(x) + x^2 B_{n-2}(x) \qquad (n \geqslant 2) ,$$

and D_0 is a constant chosen so that the attenuation at $\omega = 1$ is 3 dB.

3.5 Ultraspherical family of filters

$$1/|H(s)|^2 = 1 + \varepsilon^2 \left[Y_n^\alpha(x) \right]^2$$

where $Y_n^\alpha(x)$ is a polynomial defined recursively by

$$Y_0^\alpha(x) = 1$$

$$Y_1^\alpha(x) = x$$

$$(2\alpha + n) Y_n^\alpha(x) = x(2\alpha + 2n - 1) Y_{n-1}^\alpha(x) - (n - 1) Y_{n-2}^\alpha(x) \qquad (n \geq 2)$$

and

$$\varepsilon^2 = 10^{0.1\gamma} - 1$$

where γ is the attenuation at $\omega = 1$. The parameter α can lie in the range $-1 < \alpha < +\infty$. Any fixed value of α defines one filter type out of the Ultraspherical family of filter types.

3.6 Papoulis filter

$$1/|H(s)|^2 = 1 + T_n(\omega)$$

where

$$T_1(\omega) = \omega^2$$

$$T_2(\omega) = \omega^4$$

$$T_3(\omega) = 3\omega^6 - 3\omega^4 + \omega^2$$

$$T_4(\omega) = 6\omega^8 - 8\omega^6 + 3\omega^4$$

$$T_5(\omega) = 20\omega^{10} - 40\omega^8 + 28\omega^6 - 8\omega^4 + \omega^2$$

$$T_6(\omega) = 50\omega^{12} - 120\omega^{10} + 105\omega^8 - 40\omega^6 + 6\omega^4$$

$$T_7(\omega) = 175\omega^{14} - 525\omega^{12} + 615\omega^{10} - 355\omega^8 + 105\omega^6 - 15\omega^4 + \omega^2$$

$$T_8(\omega) = 490\omega^{16} - 1680\omega^{14} + 2310\omega^{12} - 1624\omega^{10} + 615\omega^8 - 120\omega^6 + 10\omega^4 .$$

3.7 Halpern filter

$$1/|H(s)|^2 = 1 + U_n(\omega) ,$$

where $U_1(\omega) = \omega^2$

$$U_2(\omega) = \omega^4$$

$$U_3(\omega) = 4\omega^6 - 6\omega^4 + 3\omega^2$$

$$U_4(\omega) = 9\omega^8 - 16\omega^6 + 8\omega^4$$

$$U_5(\omega) = 36\omega^{10} - 90\omega^8 + 80\omega^6 - 30\omega^4 + 5\omega^2$$

$$U_6(\omega) = 100\omega^{12} - 288\omega^{10} + 306\omega^8 - 144\omega^6 + 27\omega^4$$

$$U_7(\omega) = 400\omega^{14} - 1400\omega^{12} + 1932\omega^{10} - 1330\omega^8 + 476\omega^6 - 84\omega^4 + 7\omega^2 \quad .$$

3.8 Synchronously tuned filter

$$1/|H(s)|^2 = [1 + q\omega^2]^n$$

where $q = 2^{1/n} - 1$.

3.9 User-defined filter

In program ACFC (but not in programs ACFA, ACFD) the user may define his own filter type. The filter is specified by giving the poles of the normalised low-pass prototype transfer function. It is assumed that all the poles occur in complex conjugate pairs, with the possible exception of a single real pole.

4 CIRCUIT TYPES

In this section we describe the configurations used to realise the first and second order transfer functions. For each circuit type we give the design equations, that is, equations relating the component values to terms in the transfer function. We recall that a first order transfer function has the form

$$\frac{Ga}{s + a}$$

for low-pass, or

$$\frac{Gs}{s + a}$$

for high-pass filter. A second order transfer function has the form

$$\frac{G\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{E}{s^2 + As + B}$$

for a low-pass, or

$$\frac{Gs^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{Cs^2}{s^2 + As + B}$$

for a high-pass, or

$$\frac{G\omega_0 s/Q}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{Ds}{s^2 + As + B}$$

for a band-pass filter.

There is just one type of first order section but there are six second order sections: Unity Gain, Single Feedback, Multiple Feedback, State Variable, Ring of Three, Mitra-Aatre. These circuits are just a few out of the many that have been suggested in the literature.

In each case the transfer function is realised as a voltage ratio. On the appropriate circuit diagram the input voltage is denoted by V_{in} and the output voltage is denoted by V_{out} . Where the same configuration can be used for different types of filter V_{LP} denotes the low-pass output, V_{HP} denotes the high-pass output and V_{BP} denotes the band-pass output.

4.1 First order section[3]

This is illustrated in Fig 9 and is available in low-pass and high-pass forms. The design equation in both cases is

$$a = 1/(R_1 C_1) \quad .$$

Thus C_1 may be chosen arbitrarily, then R_1 is given by

$$R_1 = 1/(C_1 a) \quad .$$

The gain of this section is unity, *ie* $G = 1$.

4.2 Unity Gain[3]

This section type is available in low-pass or high-pass forms and is illustrated in Figs 10 and 11. The low-pass configuration uses two resistors, two capacitors and two unity gain operational amplifiers. The design equations are

038

$$A = 1/(R_1C_1)$$

$$B = 1/(R_1R_2C_1C_2)$$

$$G = 1 \quad .$$

C_1 and C_2 can be chosen arbitrarily and R_1 and R_2 can be calculated from

$$R_1 = 1/(AC_1)$$

$$R_2 = A/(BC_2) \quad .$$

The high-pass configuration uses two capacitors, two resistors and one unity gain operational amplifier. The two capacitors have the same value. The design equations are

$$A = 1/(R_2C_2) + 1/(R_2C_1)$$

$$B = 1/(R_1R_2C_1C_2)$$

$$G = 1 \quad .$$

We can choose $C_1 = C_2$ arbitrarily, then put

$$R_1 = A/(2BC_1)$$

$$R_2 = 2/(BC_1) \quad .$$

4.3 Single Feedback[4]

This circuit type is available in low-pass, high-pass or band-pass forms as shown in Figs 12, 13, 14.

The low-pass configuration has four resistors, two capacitors and one operational amplifier. The design equations are

$$A = \frac{1}{C_3}\left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$B = 1/(R_3R_4C_2C_3)$$

$$G = R_3R_4C_3/(R_1R_2C_1)$$

and we must also have

$$R_3R_4C_3/(R_1C_1) \quad = \quad (R_3 + R_4)R_2/(R_1 + R_2) \quad .$$

We can choose C_2 arbitrarily, then put

$$R_1 \quad = \quad R_2 \quad = \quad x/(2Gy)$$

$$R_3 \quad = \quad 1/(yz)$$

$$R_4 \quad = \quad (x - 1/z)/y$$

$$C_1 \quad = \quad 4GC_2/x^2$$

$$C_3 \quad = \quad z^2C_2/(xz - 1)$$

where $y \quad = \quad C_2\sqrt{B}$

$x \quad = \quad A/\sqrt{B}$

and z is an arbitrary constant satisfying $xz > 1$.

The high-pass configuration has three resistors, four capacitors and one operational amplifier. The design equations are

$$A \quad = \quad \frac{1}{C_4}\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$B \quad = \quad 1/(R_2R_3C_3C_4)$$

$$G \quad = \quad C_1C_2R_1(R_2 + R_3)/(P_2R_3C_3C_4)$$

and we must also have

$$R_1(R_2 + R_3)(C_1 + C_2) \quad = \quad R_2R_3C_4 \quad .$$

We can choose C_3 arbitrarily, then put

$$C_1 \quad = \quad C_2 \quad = \quad 2GC_3$$

$$C_4 \quad = \quad z^2C_3/(xz - 1)$$

$$R_1 \quad = \quad 1/(4Gxy)$$

$$R_2 \quad = \quad 1/(yz)$$

$$R_3 \quad = \quad (x - 1/z)/y$$

where $y = C_3\sqrt{B}$

$x = A/\sqrt{B}$

and z is an arbitrary constant satisfying $xz > 1$.

The band-pass configuration uses four resistors, four capacitors and one amplifier. Exact design equations cannot be given here but an approximate design procedure is to choose C_2 arbitrarily and then set

$$C_1 = Gy/\omega_0$$

$$C_3 = yz/\left((z - 1)\omega_0\right)$$

$$C_4 = \frac{z}{1 + x} C_3$$

$$R_1 = 1/(Gy)$$

$$R_2 = 1/(yz)$$

$$R_3 = (z - 1)/(yz)$$

$$R_4 = (x + 1)R_3/z$$

where $x = A/\sqrt{B}$

$y = C_2\omega_0/z$

and z is an arbitrary parameter such that $z > 1$.

4.4 Multiple Feedback[4]

This circuit type is available in low-pass, high-pass and band-pass forms, as shown in Figs 15, 16, 17.

The low-pass configuration uses three resistors, two capacitors and one operational amplifier. The design equations are

$$A = (1/R_1 + 1/R_2 + 1/R_3)/C_1$$

$$B = 1/(C_1C_2R_2R_3)$$

$$G = R_3/R_1 .$$

038

We can choose C_2 arbitrarily and set

$$C_1 = 4BC_2(G+1)/A^2$$

$$R_3 = A/(2BC_2)$$

$$R_1 = R_3/G$$

$$R_2 = R_3/(G+1) \; .$$

The high-pass configuration uses two resistors, three capacitors and one operational amplifier. The design equations are

$$A = (C_1 + C_2 + C_3)/(R_2C_2C_3)$$

$$B = 1/(R_1R_2C_2C_3)$$

$$G = C_1/C_3 \; .$$

We can choose $C_1 = C_2$ arbitrarily, then set

$$C_3 = C_1/G$$

$$R_1 = A/\Big(BC_1[2 + 1/G]\Big)$$

$$R_2 = (2G+1)/(AC_1) \; .$$

The band-pass configuration uses three resistors, two capacitors and one operational amplifier. The design equations are

$$A = (C_1 + C_2)/(R_3C_1C_2)$$

$$B = (1/R_1 + 1/R_2)/R_3$$

$$G = C_1/(R_1B) \; .$$

We can choose $C_1 = C_2$ arbitrarily, then set

$$R_1 = 1/(C_1GA)$$

$$R_2 = x/(2/y - GA/\sqrt{B})$$

$$R_3 = 2x/y$$

where $x = 1/(C_1\sqrt{B})$

$y = A/\sqrt{B}$.

4.5 State Variable[5]

The single circuit shown in Fig 18 realises a low-pass, high-pass or band-pass transfer function depending on whether V_{LP}, V_{HP} or V_{BP} is taken as the output voltage. The circuit uses six resistors, two capacitors and three operational amplifiers. The low-pass design equations are

$$A\tau = (1 + R_2/R_3)/(1 + R_5/R_4)$$

$$B\tau^2 = R_2/R_3$$

$$G = (1 + R_3/R_2)/(1 + R_4/R_5)$$

where $\tau = R_1C_1$.

We can choose R_3, R_4 and C_1 arbitrarily. Then, if we let $x = R_5/R_4$, $y = R_2/R_3$ we can solve the following quadratic equation for $\sqrt{y}$

$$Q(G - 1)y + \sqrt{y} - Q = 0$$

and then set

$$x = QG\sqrt{y}$$

$$R_2 = R_3y$$

$$R_5 = R_4x$$

$$R_1 = \sqrt{y}/(\omega_0C_1) \quad .$$

However, we may find that the quadratic equation has no real roots. In this case we abandon the attempt to achieve the specified gain, G , and set

$$R_2 = R_3$$

$$R_5 = (2Q - 1)R_4$$

$$R_1 = 1/(\omega_0C_1) \quad .$$

The actual gain will then be equal to $2/(1 + R_4/R_5)$.

038

The high-pass design equations are

$$A\tau = (1 + R_2/R_3)/(1 + R_5/R_4)$$

$$B\tau^2 = R_2/R_3$$

$$G = (1 + R_2/R_3)/(1 + R_4/R_5) \quad .$$

We can choose R_3, R_4 and C_1 arbitrarily. Then, letting $x = R_5/R_4$, $y = R_2/R_3$ we can solve the following equation for $\sqrt{y}$

$$Qy - \sqrt{y} + Q(G + 1) = 0 \quad .$$

We then set

$$x = QG/\sqrt{y}$$

$$R_2 = R_3 y$$

$$R_5 = R_4 x$$

$$R_1 = \sqrt{y}/(\omega_0 C_1) \quad .$$

It may be that the quadratic equation has no real roots. In this case we abandon the attempt to achieve the specified gain, G, and set

$$R_2 = R_3$$

$$R_5 = (2Q - 1)R_4$$

$$R_1 = 1/(\omega_0 C_1) \quad .$$

The actual gain will then be equal to $2/(1 + R_4/R_5)$.

The band-pass design equations are

$$A\tau = (1 + R_2/R_3)/(1 + R_5/R_4)$$

$$B\tau^2 = R_2/R_3$$

$$G = R_5/R_4 \quad .$$

038 We can choose R_3, R_4 and C_1 arbitrarily. Then letting $x = R_5/R_4$, $y = R_2/R_3$ we can solve the following quadratic equation for $\sqrt{y}$

$$Qy - (1 + G)\sqrt{y} + Q = 0 .$$

We then set

$$x = G$$

$$R_2 = R_3 y$$

$$R_5 = R_4 x$$

$$R_1 = \sqrt{y}/(\omega_0 C_1) .$$

It may be that the quadratic equation has no real roots. In this case we abandon the attempt to achieve the specified gain, G, and set

$$R_2 = R_3$$

$$R_5 = (2Q - 1)R_4$$

$$R_1 = 1/(\omega_0 C_1) .$$

The actual gain will then be $2Q/(1 + R_4/R_5)$

4.6 Ring of Three[6-8]

The circuit of Fig 19 realises a low-pass or band-pass filter depending on whether V_{LP} or V_{BP} is taken as the output. It uses six resistors, two capacitors and three amplifiers. The low-pass design equations are

$$\omega_0^2 = R_2/(R_1 R_3 R_4 C_1 C_2)$$

$$Q = R_5 C_2 \omega_0$$

$$G = R_4/R_6 .$$

We can choose R_1, R_2 and $C_1 = C_2$ arbitrarily, then set

$$R_3 = \frac{1}{C_1 \omega_0}\sqrt{\frac{R_2}{R_1}}$$

$$R_5 = Q/(C_1 \omega_0)$$

$$R_6 = R_3/G$$

$$R_4 = R_3 .$$

The band-pass design equations are

$$\omega_0^2 = R_2/(R_1 R_3 R_4 C_1 C_2)$$

$$Q = R_5 C_2 \omega_0$$

$$G = R_5/R_6 \quad .$$

We can choose R_1, R_2 and $C_1 = C_2$ arbitrarily, then set

$$R_3 = \frac{1}{C_1 \omega_0}\sqrt{\frac{R_2}{R_1}}$$

$$R_5 = Q/(C_1 \omega_0)$$

$$R_6 = R_5/G$$

$$R_4 = R_3 \quad .$$

4.7 Mitra-Aatre[9]

This circuit is available only in band-pass form and is illustrated in Fig 20. It uses no capacitors, seven resistors and two operational amplifiers. This circuit differs from all others considered in this Report in that it uses a different model of the operational amplifier. Other circuits assume an ideal amplifier, *ie* if $V_1(s)$ and $V_2(s)$ are the inverting and non-inverting inputs to the amplifier at a given frequency and $V_3(s)$ is the corresponding output then

$$V_3(s) = A_0\Big(V_2(s) - V_1(s)\Big)$$

where $A_0 = +\infty$.

The present circuit replaces this equation by

$$V_3(s) = \frac{P}{s}\Big(V_2(s) - V_1(s)\Big)$$

where P is the gain-bandwidth product of the amplifier. Like the first equation this is only an approximation to the truth, but Mitra and Aatre claim reasonable performance for centre frequencies between 5 and 200 kHz.

The design equations are

$$B = P_1 P_2 / [(1 + R_3/R_4)(1 + R_6/R_7)]$$

$$A = P_1/(1 + R_5/R_1 + R_5/R_2)$$

$$G = R_5/R_1 \quad .$$

We can choose R_1 and R_4 arbitrarily, then set

$$R_5 = R_1 G$$

$$R_2 = R_1 G/(A - G - 1)$$

$$R_3 = R_4\left(\sqrt{\frac{P_1 P_2}{B}} - 1\right)$$

$$R_6 = R_3 \ , \ R_7 = R_4$$

where P_1 and P_2 are the gain-bandwidth products of the two amplifiers. For ease of programming we have assumed that all operational amplifiers have the same gain-bandwidth product, but this is not essential.

4.8 Choice of circuit types

Since we have described six alternative second order circuit types we may ask which of these types is best. No definitive answer can be given to this question since each type has certain advantages. The Unity Gain circuit has the smallest number of passive components and has a simple structure but cannot give arbitrary gain and is not available in band-pass form. Both Single Feedback and Multiple Feedback circuits use only one amplifier per section and can realise arbitrary gain. Of the two, the Multiple Feedback uses fewer passive components, but may require a larger spread of element values in some circumstances. The Single Feedback circuit may be viewed with some suspicion since it relies on exact pole-zero cancellation in the transfer function. Both Single Feedback and Multiple Feedback types use in general at least one capacitor which cannot be constrained to take a standard value.

The State Variable and Ring of Three circuits both use three amplifiers per section. They use the same number of passive components. The Ring of Three circuit can achieve arbitrary gain, whereas the State Variable circuit can only do this in some circumstances.

The Mitra-Aatre circuit uses no capacitors but requires a knowledge of the gain-bandwidth product of the amplifiers. It is intended for use at higher frequencies, say greater than 5 kHz.

If a rule of thumb is desired we may say: for low Q sections use the Unity Gain circuit. For good sensitivity performance use the Ring of Three circuit. For high frequency-operation try the Mitra-Aatre circuit.

Program ACFC assumes that all second order sections will be of the same type, *ie* either all Unity Gain or all Single Feedback etc. But there is nothing to stop the designer mixing section types in the real circuit. For example, a sixth order filter might have two low Q sections and one high Q section. One could run program ACFC with two sets of data, one for Unity Gain sections only and one for Ring of Three sections only. One could then use the Unity Gain components for the low Q sections and the Ring of Three components for the high Q section.

As we mentioned in section 2, some section types invert their output with respect to the input. The inverting types are: all single feedback, multiple feedback, Ring of Three, Mitra-Aatre types and the high-pass State Variable. The remaining types are non-inverting, *ie* first order section, Unity Gain, low-pass and high-pass State Variable.

5 CHOICE OF COMPONENT VALUES

In all the circuits of section 4 the number of passive elements exceeds the number of design equations, so that one or more components may be chosen arbitrarily. We can use this freedom in order to achieve satisfactory values for all components. We would like the component values to have the following properties:

(i) All capacitance and resistance values should lie between lower and upper bounds specified by the user.

(ii) Only preferred capacitance values should be used.

(iii) Values should lie near the middle of the ranges specified by the user.

(iv) The spread of resistance values and the spread of capacitance values should be small.

It will not, in general, be possible to achieve all of these properties: sometimes we shall not be able to achieve any of them. Program ACFC tries to obtain satisfactory component values by an iterative process that we now describe.

038 In program ACFC arbitrary capacitances are allowed to take only the following values (in farads)

$$10^{-11}, \quad 2.2 \times 10^{-11}, \quad 5 \times 10^{-11},$$
$$10^{-10}, \quad 2.2 \times 10^{-10}, \quad 5 \times 10^{-10},$$
$$10^{-9}, \quad 2.2 \times 10^{-9}, \quad 5 \times 10^{-9},$$
$$10^{-8}, \quad 2.2 \times 10^{-8}, \quad 5 \times 10^{-8},$$
$$10^{-7}.$$

These preferred values could of course be changed as desired. For most of the circuits (First Order, Unity Gain, State Variable, Ring of Three, Mitra-Aatre) all capacitances are arbitrary, but in some (single feedback and multiple feedback) there are non-arbitrary capacitances which cannot be forced to have preferred values.

The program makes no attempt to use only preferred resistance values, but it does allow the user to specify lower and upper bounds, R_{min} and R_{max} for resistances, and also C_{min} and C_{max} for capacitances. The user can also provide a single preferred value for any arbitrary resistance, though if this choice leads to other components taking unsatisfactory values it may be changed.

For any first or second order section the following procedure is carried out:

(i) Set all arbitrary capacitances to a preferred value in the middle of the range, namely 10^{-9} farads.

(ii) Set all arbitrary resistances to the preferred value supplied by the user.

(iii) Calculate the other component values from the equations given in section 4.

(iv) Check whether any calculated component values lie outside their bounds. If so change one of the arbitrary values. If an arbitrary capacitance is to be changed, select the next lower or next higher preferred value.

(v) Go to step (iii).

The procedure is continued until either a satisfactory set of components is obtained or it is clear that no such set can be obtained. If a satisfactory set cannot be found an appropriate message is printed, but the program does not halt:

038

it uses the last calculated set of values. Having calculated the component values we may wish to change them prior to performing a sensitivity or performance analysis, for example if we wish to investigate the effect of using only standard value resistances. The program allows the calculated values to be overwritten in this way.

We have not yet described in detail how arbitrary component values are changed to avoid having components lying outside their bounds. Appendix B gives the algorithm used for the Ring of Three low-pass circuit. Similar algorithms have been derived for the other circuit types.

6 SENSITIVITY

It is of interest to determine how the performance of a circuit changes as the value of some component changes. Let r denote the variable measuring the performance and p denote the parameter being changed. We define the differential relative sensitivity to be

$$\begin{aligned} S_p^r &= \lim_{\Delta p \to 0} (\Delta r/r)/(\Delta p/p) \\ &= \frac{p}{r}\,\frac{\partial r}{\partial p} \, . \end{aligned}$$

We are mostly interested in the gain of the filter which is equal to the absolute value of the transfer function. It is shown in Appendix C that the required sensitivity can be expressed in terms of the quantities

$$p, \omega_0, Q, G, \frac{\partial \omega_0}{\partial p}, \frac{\partial Q}{\partial p}, \frac{\partial G}{\partial p} \, .$$

To evaluate the partial derivatives we need to consider the design equations of the particular circuit. For example, for the Ring of Three circuit,

$$\omega_0^2 = R_2/(R_1 R_3 R_4 C_1 C_2)$$

so

$$\frac{\partial \omega_0}{\partial R_1} = -\,0.5\omega_0/R_1$$

$$\frac{\partial \omega_0}{\partial R_2} = 0.5\omega_0/R_2$$

etc.

Having found the differential sensitivities we can estimate the tolerances required for each passive component. For suppose that M is the maximum allowable relative change in gain, *ie* we require that $\frac{\Delta|H|}{|H|} \leq M$. Then since

$$S_p^{|H|} \simeq \frac{p}{|H|}\frac{\Delta|H|}{\Delta p} \qquad \text{we have} \qquad \frac{\Delta p}{p} \leq \frac{M}{S_p^{|H|}} ,$$

so that $M/S_p^{|H|}$ is an estimate for the required tolerance of parameter p .

The differential sensitivity $S_p^{|H|}$ and the corresponding tolerance figure give a useful indication of how the performance will be changed by small changes in the parameter. However these figures should not be given too much importance since they assume infinitesimal changes in the parameters and also assume that only one parameter is changing at a time, thus ignoring coupling effects between components. For these reasons it is worth calculating also large change sensitivities, by which we mean relative changes in gain produced by specified changes in one or more components. The computer program allows the user to increase or decrease any or all of the components by arbitrary amounts and calculates the corresponding gain from the design equations of the appropriate section type. The program does not however perform any automatic or 'Monte Carlo' calculations to establish the effect of random perturbations of the parameters. It is left to the user to decide which sets of parameter changes to investigate.

Program ACFC performs sensitivity calculations for only two filter types (Unity Gain and Ring of Three), but there is no reason in principle why other filter types should not be included.

7 PERFORMANCE OF CIRCUIT

If the design has been correctly carried out it would seem that the circuit should behave exactly as desired. There are a number of reasons why this might not be true. The component values as calculated from the equations of section 4 may have been overwritten as described in section 5, for example if resistances have been rounded to convenient values. Also, the operational amplifiers may have been incorrectly modelled. The equations of section 4 assume that the operational amplifiers have infinite input impedances and zero output impedances. The equations for all section types except the Mitra-Aatre assume the amplifiers have infinite open-loop gain at all frequencies. Program ACFC allows the user to specify values of input and output impedances and open-loop gain, the same values being assumed to apply to all the amplifiers in the circuit. The program will then calculate, for any frequency specified by the user, 038

(i) the input and output impedances and gain of each section,

(ii) the minimum and maximum voltages occurring in the circuit, and the voltage spread, (*ie* the ratio of maximum to minimum voltage),

(iii) the transmission matrix of the complete circuit,

(iv) the input and output impedances and gain of the complete circuit.

This should help the user to make a good estimate of the circuit performance, but it will not enable him to predict exactly the actual circuit performance since there are many factors not taken into account by the program. For example, even with realistic input and output impedances and open-loop gain the model of the operational amplifier is crude - it does not include effects due to frequency - dependent gain, slew rate, output clipping etc.

We now discuss the calculation of the quantities (i) to (iv) above. The zero output impedance of the ideal operational amplifier means that each section can be considered independently of the rest of the circuit. When non-ideal amplifiers are considered however, the different sections will interact. One consequence of this is that the ordering of the sections makes a difference and must be specified. If the user is not sure which ordering to use he may either try different orderings and see which gives the best results or use the rule of thumb that the sections should be arranged in order of increasing Q . Another consequence is that we are unable to derive a voltage transfer function since we do not know how the network is terminated. The best we can do is to derive the transmission matrix[10] of the network, *ie* the matrix $\mathscr{A}$ relating input to output quantities as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathscr{A} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where suffix 1 denotes input quantities and suffix 2 denotes output quantities (the positive inwards current convention is assumed here). Each section has an associated transmission matrix and since the sections are connected in cascade, the transmission matrix of the network is the product of the transmission matrices of the individual sections. Once we have the transmission matrix it is easy to calculate the input and output impedances and gain. If

$$\mathscr{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

then

$$\text{the input impedance} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{a_{11}}{a_{21}}$$

$$\text{the output impedance} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{a_{22}}{a_{21}}$$

and

$$\text{the gain} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{1}{a_{11}} \ .$$

(NB. The transmission matrix elements are usually denoted by A, B, C, D where $A = a_{11}$, $B = a_{12}$, $C = a_{21}$, $D = a_{22}$. We have avoided this notation since we have reserved the letters A, B, C, D for other uses).

The voltage spread can be estimated as follows. The voltage and current at the end of the last section are set arbitrarily to 1 and 0 respectively. Pre-multiplication by the transmission matrix of the last section gives the voltage and current at the beginning of the last section. Multiplication by each matrix in turn thus gives the voltages at the end of each section. We can then pick out the largest and smallest of these voltages and divide the largest by the smallest to give the voltage spread. It should be remembered that these figures are based on unit voltage and zero current at the end of the last section. It should also be noted that the voltages inside each section are not calculated. It is possible that such a voltage is greater than the largest of the inter-section voltages. Thus the voltage spread as calculated is only a lower bound for the true voltage spread.

The problem of finding the transmission matrix of a single section is discussed in Appendix D. The calculations will depend on the section type, but as an example, the Appendix gives the procedure for the low-pass Ring of Three section.

So, having determined the circuit component values, quantities (i) to (iv) can be found. This assumes that we have decided on the order in which the sections are connected. For the calculation of component values and sensitivities the ordering of the sections is irrelevant since the operational amplifiers are assumed to be ideal for these calculations. When realistic input and output impedances are used, however, the ordering does make a difference, and must be specified. There is no simple procedure for choosing the best ordering, but a

038

rule of thumb that is sometimes used is to arrange the sections in order of increasing Q . Program ACFC allows the user to specify the ordering or to use this rule of thumb.

In this section, we have described how various performance characteristics of the circuit (items (i) to (iv) above) can be calculated for any circuit type. Program ACFC allows these characteristics to be calculated for only two circuit types (Unity Gain and Ring of Three), but there is no reason in principle why the same calculations should not be performed for any other circuit type.

8 GENERAL NOTES ON THE PROGRAMS

In this section we give notes on programs ACFA, ACFD and ACFC and on their use on the RAE ICL 1906S computer.

The programs are written in Algol 68-R and are designed to be run under the George 4 operating system. The following files are contained in directory :RN4660G.

ACFA, ACFC, ACFD, ACFSEG1, ACFSEG2, ACFSEG3, ACFALB, ACFALBUM, ACFTIDY.

Files ACFA, ACFC and ACFD contain source programs. ACFSEG1, ACFSEG2, ACFSEG3 contain source procedures used by these programs. Files ACFALB and ACFALBUM are Algol 68-R albums containing compiled versions of the procedures in ACFSEG1, ACFSEG2 and ACFSEG3. ACFTIDY is a macro used to edit the output produced by ACFC. Copies of card listings of these files may be obtained from the author.

The three programs may be run in either interactive or non-interactive mode. In interactive mode, the programs are run from a MOP console and the user inputs data in response to questions output by the programs to the console. In non-interactive mode the data are first stored in a file, the program is then run either at a MOP console or as a background job, the results being stored in a file.

The commands for running program ACFA in non-interactive mode are

```
COMPILE :RN4660G. ACFA
AS *TRO, file 1
AS *LPO, file 2
EXECUTE
```

where file 1 is the file containing the data and file 2 is the file which is to contain the output. If running in interactive mode the two assignment commands are omitted, so the commands are simply

```
COMPILE :RN4660G. ACFA
EXECUTE.
```

The commands for running ACFD are similar. They are

```
COMPILE :RN4660G. ACFD
AS *TRO, file 1 } (if in non-interactive mode)
AS *LPO, file 2 }
EXECUTE.
```

The commands for running ACFC are similar except that an extra command is needed after compilation

```
COMPILE :RN4660G. ACFC
GETEBM PRθGC
AS *TRO, file 1 } (if in non-interactive mode)
AS *LPO, file 2 }
EXECUTE.
```

If program ACFC is run in non-interactive mode the output file will contain the questions which would have been asked at the console if the mode were interactive. Since these questions are of no interest to the user after the program has been run, a macro, ACFTIDY, has been written to edit out the questions from the output file. The macro command is

```
ACFTIDY  file 2
```

if the job is being run under directory :RN4660G or

```
OBEY :RN4660G. ACFTIDY, PARAM (file 2)
```

if the job is being run under another directory. The macro will generate two files named LJRTEM1 and LJRTEM2 and then erase them.

Normally the programs will read all their data and calculate all the required results. They may fail however if either the restrictions given in sections 9.3, 10.3, 11.3 are not observed, or if the transfer function cannot be obtained. In a number of cases to obtain the transfer function the program needs to find the roots of a polynomial equation. If, particularly for higher orders, these roots cannot be found an appropriate message is printed and the program will be halted.

Program ACFD automatically calls up and makes use of album ACFALB and program ACFC uses the two albums ACFALB and ACFALBUM. The user need not normally

concern himself with these two albums or with the source files ACFSEG1, ACFSEG2, ACFSEG3 from which they are obtained. However, for reference we give the procedure for setting up the two albums from the three source files. To set up ACFALB the following commands are required

```
CE ACFALB (*ED, BUCKET1, KWθRDS 19)
COMPILE ACFSEG1
UPDATEALBUM ACFALB
CLEARTθ :0
PUTIN SEG1
END
COMPILE ACFSEG2
UPDATEALBUM ACFALB
PUTIN SEG2
END.
```

To set up ACFALBUM one should first set up ACFALB and then issue the following commands

```
CE ACFALBUM (*ED, BUCKET1, KWθRDS 20)
COMPILE ACFSEG3
UPDATEALBUM ACFALBUM
CLEARTθ :ACFALB
PUTIN SEG3
END.
```

Also for reference, we point out that if the albums are set up under a directory other than :RN4660G then the first line of some of the files will need to be changed. For example the first line of file ACFD is at present

```
PROG1B 'WITH' SEG1, SEG2 'FROM' :RN4660G. ACFALB.
```

If the albums to be used are :ABC123. ACFALB and :ABC123. ACFALBUM then the first line of ACFD should be changed to

```
PROG1B 'WITH' SEG1, SEG2 'FROM' :ABC123. ACFALB.
```

Similar changes will be required for files ACFC, ACFSEG2 and ACFSEG3.

For users more familiar with FORTRAN than Algol 68-R it should be observed that in input and output the symbol denoting exponent of 10 is the ampersand (&) rather than the E of FORTRAN. Thus the number 0.0012 could be written as 1.2E-3 in FORTRAN but would be 1.2&-3 in Algol 68-R. It should also be noted that a real number should contain a decimal point and at least one digit

following the decimal point. In FORTRAN a number could be input in the form 12E-4 or 12.E-4 but in Algol 68-R the number would have to be given as 12.0&-4. On output it should be noted that Algol 68-R outputs a complex number in the form

real part ? imaginary part.

9 PROGRAM ACFA

This program calculates the attenuation performance of a given filter. The filter type is chosen from the list given in section 3. The user can either specify the order of the filter and let the program calculate the attenuation at specified frequencies of interest or let the program calculate the minimum order needed to achieve a specified attenuation at a specified frequency.

9.1 Input parameters

The following input parameters are required in the following order. Unless stated otherwise there should be one item per record.

(1) One of the three symbols L, H, B, depending on whether the filter is low-pass, high-pass or band-pass. The symbol should be the first character of the record.

(2) Filter name. This should begin at the first character of the record. Only the first four characters of the name are significant. Thus a Gaussian filter could be specified by either GAUSSIAN or GAUS.

(3) Alpha. This real number applies only to Ultraspherical filters (see section 3.5) and selects one filter type out of the Ultraspherical family of filter types. For other filter types this parameter should be omitted.

(4) The order of the filter. If this is given as zero the minimum order required will be calculated by the program. In the case of band-pass filters the order to be given is the order of the low-pass prototype (*ie* half the actual order).

(5) The attenuation in dB required at a given stop-band frequency. This parameter should only be given if the filter order was given as zero.

(6) The cut-off frequency in hertz. This parameter should only be given if the filter is low-pass or high-pass.

(7) Geometric centre frequency and bandwidth, both in hertz. These numbers should appear on the same record. The parameters should only be given if the filter is band-pass.

038

(8) The attenuation in dB at the cut-off frequency (or frequencies).

(9) The number of frequencies of interest. This is an integer not greater than 20.

(10) The frequencies of interest, in hertz. If the order of the filter was given as zero the first frequency given will be assumed to be the frequency at which the attenuation was specified. The numbers may be spread over any number of records.

9.2 Program output

The attenuation at each frequency of interest is output in the form

ATTENUATION AT FREQ. no. WITH ORDER no. IS no.

The order referred to is the order of the low-pass prototype which in the case of band-pass filters is half the actual order. To emphasize this, in the band-pass case the actual order of the band-pass filter is output after the last frequency of interest. If the program is required to find the minimum order needed to achieve a given attenuation at a given frequency then the attenuation at all frequencies of interest will be output for orders 1, 2, 3 ..., the order being increased until either the required attenuation is achieved or the order exceeds the maximum order allowable for the filter type (see section 9.3 below).

9.3 Restrictions

The order of the filter types are restricted as shown in the following table.

Filter type	Maximum order
Butterworth	30
Chebyshev	20
Ultraspherical	20
Papoulis	8
Halpern	7
Gaussian	20
Bessel	6
Synchronously Tuned	20

In the case of band-pass filters the order referred to in this table is that of the low-pass prototype.

As stated previously there can be not more than 20 frequencies of interest. If it is desired to change this limit to, say, 50, then the tenth line of program ACFA should be changed from

038

[1 : 20] 'REAL' WINT ;

to

[1 : 50] 'REAL' WINT ;

9.4 Examples of use

(1) A fourth order high-pass Bessel filter with cut-off frequency 1500 Hz is to be considered. The attenuations at frequencies 1200 Hz, 1000 Hz and 800 Hz are required. Fig 1 shows the results of an interactive run of program ACFA.

(2) It is required to construct a Chebyshev band-pass filter with centre frequency 500 Hz and bandwidth 5 Hz. A ripple of 0.1 dB is allowed in the pass-band. An attenuation of at least 70 dB is required at 400 Hz. The results of an interactive run of program ACFA are shown in Fig 2.

10 PROGRAM ACFD

This program calculates the delay performance of a given filter. The user specifies the filter order and filter type and the program calculates the delay at specified frequencies of interest.

10.1 Input parameters

The following input parameters are required in the following order. Unless stated otherwise there should be one item per record.

(1) One of the three symbols L, H, B, depending on whether the filter is low-pass, high-pass or band-pass. The symbol should be the first character of the record.

(2) Filter name. This should begin at the first character of the record. Only the first four characters of the name are significant, so that a Gaussian filter, for example, could be specified by either GAUSSIAN or GAUS.

(3) Alpha. This real number applies only to Ultraspherical filters (see section 3.5) and selects one filter type out of the Ultraspherical family of filter types. For other filter types this parameter should be omitted.

(4) The order of the filter. In the case of band-pass filters the order to be given is the order of the low-pass prototype (*ie* half the actual order).

(5) The cut-off frequency in hertz. This parameter should only be given if the filter is low-pass or high-pass.

(6) Geometric centre frequency and bandwidth, both in hertz. These numbers should appear on the same record. The parameters should only be given if the filter is band-pass.

(7) The attenuation in dB at the cut-off frequency (or frequencies).

(8) The number of frequencies of interest. This is an integer not greater than 20.

(9) The frequencies of interest, in hertz. The numbers may be spread over any number of records.

10.2 Program output

The delay at each frequency of interest is output in the form

DELAY AT FREQ. no. WITH ORDER no. IS no.

The delay is given in seconds. The order referred to is the order of the low-pass prototype which in the case of band-pass filters is half the actual order. To emphasize this, in the band-pass case the actual order of the band-pass filter is output after the last frequency of interest.

10.3 Restrictions

Papoulis, Halpern and Synchronously Tuned filter types are not available. The orders of the filter types are restricted as shown in the following table.

Filter type	Maximum order
Butterworth	30
Chebyshev	20
Ultraspherical	20
Gaussian	8
Bessel	6

As stated previously there can be not more than 20 frequencies of interest. If it is desired to change this limit to, say, 50, then the ninth line of program ACFD should be changed from

```
[1 : 20] 'REAL' WINT ;
```

to

```
[1 : 50] 'REAL' WINT ;
```

10.4 Example of use

Consider a low-pass fifth order Chebyshev filter with cut-off frequency 1 kHz and pass-band attenuation ripple of 0.28 dB. Fig 3 shows the results of an interactive run of program ACFD, the frequencies of interest being 100 Hz, 500 Hz, 900 Hz, 1 kHz, 1.4 kHz.

11 PROGRAM ACFC

This program calculates the capacitor and resistor values needed to realise a given filter type with a given circuit configuration. The component values are chosen where possible to have certain desirable properties, as described in section 5. For some circuits analysis of sensitivity or circuit performance can be carried out, as described in sections 6 and 7.

11.1 Input parameters

There are many options available to the user of program ACFC. There is a choice of filter types, a choice of circuit types, a choice of sensitivity calculations and so on: the data required will depend on which of the options is taken. For this reason the input requirements have been illustrated in the flowchart of Fig 22. The flowchart indicates input only - it does not show calculation or output. Rectangles indicate information to be input, each rectangle corresponding to one record unless stated otherwise below. Diamond shaped boxes refer to decisions which control the path through the flowchart. Since some of the input consists of answers to questions it is necessary to be clear about the distinction between a question to be answered in the data (rectangle) and a question to be answered by the person following the flowchart (diamond). A rectangle containing a question mark indicates that the data must start with either Y (for YES) or N (for NO). A diamond shaped box containing only a question mark stands for the question appearing in the immediately preceding box. The numbers appearing beside the boxes refer to the notes below.

(1) These are four integers. The filter order is that of the low-pass prototype, *ie* the actual order if low-pass or high-pass but half the actual order if band-pass.

Filter type	= 1	for a Butterworth filter
	= 2	for a Chebyshev filter
	= 3	for a Gaussian filter
	= 4	for a Bessel filter
	= 5	for a Ultraspherical filter
	= 0	for a Special filter defined by user

Circuit type	=	1	for Unity Gain section
	=	2	for Single Feedback section
	=	3	for Multiple Feedback section
	=	4	for State Variable section
	=	5	for Ring of Three section
	=	6	for Mitra-Aatre section
LHB	=	1	for low-pass
	=	2	for high-pass
	=	3	for band-pass.

(2) This facility is for use when considering a filter which has been normalised to have unity delay at zero frequency. The user can then de-normalise by specifying the required delay at zero frequency. If the filter is not of this type then the zero frequency delay should be set to unity.

(3) The filter is defined by the poles of its transfer function. For complex poles the real part should be given, followed by the imaginary part. The complex poles will of course occur in complex conjugate pairs: only one member of each pair should be given.

(4) The order is the order of the low-pass prototype.

(5) Alpha is the parameter which specified one particular filter type out of the Ultraspherical family of filter types.

(6) This is the overall gain desired for the whole circuit and is given as a numerical value (not in dB).

(7) These frequencies should be given in hertz.

(8) The pass-band ripple should be given in dB.

(9) A convenient value of resistance is given in ohms.

(10) These are the minimum and maximum values of capacitance (in farads).

(11) These are the minimum and maximum values of resistance (in ohms).

(12) The gain-bandwidth product of the operational amplifier is given in hertz. It is assumed to be the same for all the amplifiers in the circuit.

(13) The overall gain has been given earlier, but here the relative gain of each second order section is given. (The first order section, if present, is not involved as it always has unity gain.) The user specifies the relative gains

$$G_1, G_2, G_3, \ldots, G_m$$

where m is the number of second order sections and is equal to the largest integer less than or equal to half the actial filter order. $G_1, \ldots, G_m$ must satisfy

$$\prod_{i=1}^{m} G_i = 1 \quad .$$

If the user does not wish to specify these relative gains they will each be set to unity.

(14) After the component values have been calculated the user may change them (*eg* to investigate the effect of using preferred value resistors). Once they have been changed they will remain changed and will keep their new values until they are changed again. All future calculations, including sensitivity calculations, will be done with the new values.

(15) Some or all of the elements may be changed. Specify the number of changes.

(16) For each element to be changed one record should be input containing three quantities:

(i) the code identifying the element within its section. This is either C (capacitor) or R (resistor) followed immediately by the number of the element in the section,

(ii) the section number,

(iii) the new value for the element, in ohms or farads, *eg*

C2	3	2.3&-9
R4	1	2000.0 .

(17) Answer YES if either differential or large change sensitivities are required.

(18) Answer YES to obtain differential sensitivities of each element, considered separately, at two critical frequencies. The two critical frequencies are zero and cut-off frequency (for low-pass filters), infinite and cut-off frequencies (for high-pass filters), centre and lower cut-off frequencies (for band-pass filters).

(19) Give maximum percentage change in gain desired, so that the program may calculate the tolerances required to achieve this aim. If the user is uncertain about the value or uninterested in the result, 1.0 should be input.

038

(20) The simplest way to investigate sensitivity is to change some or all of the element values and see the change in gain at given frequencies. The user should specify the number of elements to be changed. If no elements are to be changed, zero should be input. It should be noted that contrary to the situation in item 14, the changes made here are temporary. After the large change sensitivities have been calculated, all elements revert to their previous values.

(21) For each element whose value is to be changed a record should be input containing three items:

(i) the code identifying the element within its section. This consists of either C (capacitor) or R (resistor) followed immediately by the number of the element in the section,

(ii) the section number,

(iii) the percentage change to be made in the element value. A percentage increase is assumed if the number given is positive, a percentage decrease is assumed if the number given is negative, *eg*

C2	3	5.0
R5	1	-1.0 .

(22) Answer YES to obtain the following information:

(i) input and output impedances of each section, and of the complete circuit,

(ii) the minimum and maximum voltages and voltage spread,

(iii) the transmission matrix of the complete circuit,

(iv) the actual voltage gain of the circuit when using non-ideal operational amplifiers.

(23) For the calculation of element values and sensitivities the ordering of the sections is irrelevant. For the calculation of the information referred to in item (22) however, this ordering must be known. Answer YES to this question if the sections are to be arranged in increasing order of quality factor, Q .

(24) The order of the sections should be specified, including the first order section, if present, *eg*

5 1 3 2 4 .

038

(25) The input and output impedances of the operational amplifier should be given in ohms and the open-loop dc gain as a numerical value (not in dB). The output impedance should not be given the value zero. The same parameters are assumed to apply to all the amplifiers.

(26) The information of item (22) will be calculated at the two critical frequencies defined in item (18). If this information is required at other frequencies the number of such extra frequencies should be input. If no extra frequencies are desired, input zero.

(27) Give extra frequencies in hertz.

(28) Answer YES to obtain either sensitivity or performance analysis of item (22).

(29) Answer YES to change either the relative gains (item (13)) or the component values (item (14)).

(30) Answer YES to consider a new set of filter data.

11.2 Program output

The program output may be conveniently divided into four sections: transfer function, component values, sensitivity, performance. The transfer function information consists of the poles of each first and second order second. For each section there is printed A and B where the denominator of the transfer function of that section is

$$s^2 + As + B .$$

Then the quality factor Q , and the resonant frequency ω_0 , are printed ($\omega_0^2 = B$, $\omega_0/Q = A$). For a first order section, a is printed where the denominator of the transfer function of the section is s + a . The transfer function information is given both for the normalised transfer function of the prototype low-pass filter (under the heading NORMALISED ROOTS) and for the de-normalised transfer function of the actual filter (under the heading DE-NORMALISED ROOTS).

The component value information begins with the name of the section type. For each section in turn there is output the results of the iterative process to find satisfactory component values for that section. At the end of each iteration the program prints out the values of the capacitances and resistances generated in that iteration. If no satisfactory set of values can be found for a section a message is printed to that effect. After the iteration results the final values found for each section are output under the heading COMPONENT VALUES, 038

the values being in farads and ohms. An indication is then given of whether it was found possible to achieve the desired overall gain. The message is OKGAIN = T if it was possible, or OKGAIN = F if it was not possible. It should be noted that the message OKGAIN = F will always be printed if the unity gain section is in use, even if the desired gain was in fact unity. If the desired gain was not achieved, the actual gain of each second order section and of the complete circuit will be printed.

If differential sensitivities were requested they are printed out for each section at each of the two critical frequencies (see item (18) of section 11.1). Also printed are the tolerances required for the specified percentage variation in gain at the critical frequencies. Some elements have zero sensitivities and consequently infinite tolerances. In this case the tolerance is truncated to 10^{10} × specified variation in gain. If any large change sensitivities were requested the resultant percentage change in gain at the two critical frequencies are printed. A positive value signifies an increase and a negative value signifies a decrease.

If a performance analysis was requested (see item (22) of section 11.1) then the following information is output. First, if the sections were required to be ordered with increasing quality factor then this order is printed. Then for the two critical frequencies and for any extra frequencies requested, the program outputs the absolute values of the input and output impedances of each section, the minimum and maximum voltage and voltage spread, the transmission matrix of the complete circuit, the modulus and phase of the input and output impedances of the complete circuit and the modulus and phase of the voltage gain of the complete circuit.

Whenever a section number is referred to in the output (or the input) it generally refers to the number as printed in the component values information under the heading COMPONENT VALUES. There is one exception to this rule. The maximum and minimum voltages are given in the performance information by messages of the form

MINIMUM VOLTAGE (AT END OF SECTION nn) IS xxx
MAXIMUM VOLTAGE (AT END OF SECTION nn) IS xxx .

Here nn refers to the number relative to the current ordering. For example, if there are four sections which have been put in the order

4 1 3 2

then "END OF SECTION 2" refers to the end of the second section in the new ordering, ie that originally numbered 1. Note that "END OF SECTION 0" is printed instead of "START OF SECTION 1".

As mentioned in section 8, the output will also contain the questions asked by the program to elicit the data from the user in interactive mode. To remove these questions from an output file the macro ACFTIDY can be used.

11.3 Restrictions

Only the following filter types are available: Butterworth, Chebyshev, Ultraspherical, Gaussian, Bessel, User-defined. The orders available for each filter type are restricted as indicated in the following table.

Filter type	Maximum order
Butterworth	30
Chebyshev	20
Ultraspherical	20
Gaussian	8
Bessel	6
User-defined	20 .

Any of the circuit types of section 4 can be chosen though it should be noted that not all of the types are available in low-pass, high-pass and band-pass form. The following table shows which combinations are available

	Low-pass	High-pass	Band-pass
Unity Gain	✓	✓	
Single Feedback	✓	✓	✓
Multiple Feedback	✓	✓	✓
State Variable	✓	✓	✓
Ring of Three	✓		✓
Mitra-Aatre			✓

Sensitivity and performance information is available only for Unity Gain and Ring of Three circuit types.

The standard capacitor values used are listed in section 5. It may be desired to change these values. Program ACFC can be edited to accomplish this, in the following way. If more than 20 standard values are required, then the eleventh line of the program should be changed from

```
[1 : 20] 'REAL' CASTAN ;
```

to

```
[1 : nn] 'REAL' CASTAN ;
```

where nn is the number of standard values required. The following seven consecutive lines control the setting up of the standard values in the array CASTAN.

```
NCASTAN :=13 ;
'FOR' I 'TO' 4 'DO' 'BEGIN' 'FOR' J 'TO' 3 'DO' 'BEGIN'
'REAL' TM1, TM2 ; TM1 := 'CASE' I 'IN' 1.0&-11, 1.0&-10, 1.0&-9, 1.0&-8
                                                              'ESAC' ;
TM2 := 'CASE' J 'IN' 1.0, 2.2, 5.0 'ESAC' ;
CASTAN[3*(I-1) + J]  := TM1 * TM2
'END' 'END' ;
CASTAN[13] := 1.0&-7 .
```

These lines may be changed or replaced as required.

11.4 Examples of use

Consider a twelfth order band-pass Bessel filter with centre frequency 500 Hz and bandwidth 20 Hz. The overall gain is desired to be unity at the centre frequency. A Ring of Three implementation is required with capacitances between 10 pF and 0.1 μF and resistances between 1 kΩ and 1 MΩ. A convenient arbitrary resistance value is 10 kΩ. Differential sensitivities are required, with tolerances to be calculated for a maximum 10% change in gain. Fig 4 illustrates a non-interactive run of program ACFC, the data being shown in Fig 5, and the results in Fig 6. It can be seen that four iterations per section are required to find satisfactory component values, the arbitrary capacitance being reduced step by step until all the resistances are less than 1 MΩ. Two of the resistances in each section have the given arbitrary value of 10 kΩ, but the other resistances will have to be rounded to convenient values. It may be anticipated from the differential sensitivities that this rounding will have most effect in sections 3 and 6, the resistors R_1, R_2, R_3, R_4 being particularly sensitive.

Fig 7 shows an interactive run of program ACFC. In this case we are concerned with a fourth order Butterworth low-pass filter of cut-off frequency 200 Hz. Again a Ring of Three implementation is required with resistance and capacitance bounds as in the previous example. The component values are found in one iteration for the first section and two iterations for the second section.

On this occasion we do not ask for differential sensitivities, but we investigate the effect of using 5% resistors and 2% capacitors for the first section. We find that we might obtain a decrease in gain of almost 5%. Assuming typical values of 1 MΩ for the input impedance, 300 Ω for the output impedance and 100 dB for the open-loop gain of the operational amplifier we find that the input impedance of the complete circuit is about 800 kΩ and the output impedance is of the order of milliohms. The gains at zero and cut-off frequency are approximately 0 dB and -3 dB, as required. We then decide to do another large change sensitivity calculation, this time involving only the second section. We see that a decrease in gain of 2% is predicted. Of course, before concluding that the second section is less sensitive than the first we should try changing the components in different senses and also look at the differential sensitivities. But we decide that we have enough information, and terminate the run.

12 CONCLUSIONS

We have described how a low-pass, high-pass or band-pass filter may be designed by first using program ACFA to ensure satisfactory attenuation performance, then checking the delay performance, if necessary, using program ACFD, and lastly obtaining component values from program ACFC. For some circuit types, program ACFC can then be used to investigate the sensitivity of individual components or groups of components and check the circuit performance with non-ideal operational amplifiers. We have described how these further calculations are performed for the circuit types in question and also how they may be performed for other circuit types.

Appendix A

GAIN AND FREQUENCY TRANSFORMATIONS

(see section 2)

A.1 To change the gain from unity to G, say, replace the term

$$\frac{C_k s^2 + D_k s + E_k}{s^2 + A_k s + B_k}$$

by

$$\frac{G_k C_k s^2 + G_k D_k s + G_k E_k}{s^2 + A_k s + B_k}$$

where G_1, G_2, G_3 ... are such that

$$\prod_{k=1}^{n/2} G_k = G$$

if n is even, or

$$\prod_{k=1}^{(n-1)/2} G_k = G$$

if n is odd.

A.2 To generate a low-pass filter of cut-off frequency W radians per second replace s by s/W, *ie* replace

$$\frac{e}{s + a} \quad \text{by} \quad \frac{eW}{s + aW}$$

and replace

$$\frac{E_k}{s^2 + A_k s + B_k} \quad \text{by} \quad \frac{E_k W^2}{s^2 + A_k W s + B_k W^2} .$$

A.3 To generate a high-pass filter of cut-off frequency W radians per second replace s by W/s, *ie* replace

$$\frac{e}{s + a} \quad \text{by} \quad \frac{es/a}{s + W/a}$$

and replace

$$\frac{E_k}{s^2 + A_k s + B_k} \qquad \text{by} \qquad \frac{E_k/B_k}{s^2 + A_k Ws/B_k + W^2/B_k} .$$

A.4 To generate a band-pass filter of lower and upper cut-off frequencies W_A and W_B replace s by

$$\frac{s^2 + W_A W_B}{(W_B - W_A)s} .$$

This will generate a transfer function which has order double that of the low-pass prototype and is geometrically symmetric about its centre frequency

$$\sqrt{W_A W_B} .$$

Appendix B

ALGORITHM FOR SATISFACTORY COMPONENT VALUES

(see section 5)

From section 4.6 the design equations for the Ring of Three low-pass circuit are

$$\omega^2 = R_2/(R_1R_3R_4C_1C_2)$$

$$Q = R_5C_2\omega_0$$

$$G = R_4/R_6 \quad .$$

Having set R_1, R_2 and $C_1 = C_2$ we calculate

$$R_3 = \frac{1}{C_1\omega_0}\sqrt{\frac{R_2}{R_1}}$$

$$R_5 = Q/(C_1\omega_0)$$

$$R_6 = R_3/G$$

$$R_4 = R_3 \quad .$$

Suppose we find that $R_3 > R_{max}$. We ask: can we reduce R_3 by changing one of the arbitrary resistances? The answer is that we can, by suitably re-setting R_2 . In order not to change R_2 too much from its previous value we set $R_3 = R_{max}$ and calculate the corresponding value of R_2 , which is $C_1^2\omega_0^2R_{max}^2R_1$. Now suppose instead that $R_6 > R_{max}$. Again we can change R_6 by changing R_2 , and the value of R_2 which makes $R_6 = R_{max}$ is $C_1^2\omega_0^2R_{max}^2GR_1$. To allow for the possibility that $R_3 > R_{max}$ and $R_6 > R_{max}$ we set R_2 to $C_1^2\omega_0^2x^2R_1$ where $x = \min(R_{max}, R_{max}G)$. Similarly if $R_3 < R_{min}$ or $R_6 < R_{min}$ we calculate the value of R_2 necessary to make $R_3 = R_{min}$ or $R_6 = R_{min}$.

Now suppose that $R_5 > R_{max}$. In this case we cannot change R_5 without changing C_1 . From the design equations we see that in order to reduce R_5 we must increase C_1 . This is not the only reason for which we might want to increase C_1 . If R_2 has been reduced because R_3 or R_6 were out of bounds as described above, then it might be that the new value of R_2 was less than R_{min} . Then C_1 should be increased so that in the next iteration R_3 and R_6 will be decreased. Also, we must check that C_1 has not become less than C_{max}.

Thus we must increase C_1 to the next higher preferred value if

$$R_5 > R_{max} \quad \text{or} \quad R_2 < R_{min} \quad \text{or} \quad C_1 < C_{min} \ .$$

Similarly we must decrease C_1 if

$$R_5 < R_{min} \quad \text{or} \quad R_2 > R_{max} \quad \text{or} \quad C_1 > C_{max} \ .$$

If C_1 need not be changed then we have succeeded in calculating a satisfactory set of component values for this section. If C_1 is changed, then we re-set R_1 and R_2 to the preferred value and repeat the procedure. We can fail to obtain a satisfactory set of component values for any of the following reasons.

(i) We may be unable to increase C_1 because that would make $C_1 > C_{max}$ (or unable to reduce it because that would make $C_1 < C_{min}$) .

(ii) Our algorithm may tell us to increase and decrease C_1 simultaneously *eg* if we have both $R_5 > R_{max}$ and $R_2 > R_{max}$.

(iii) Suppose at a certain iteration we decide to increase C_1 and at the next iteration we decide to decrease C_1 . Then, unless the situation were detected, the algorithm would oscillate forever between two preferred capacitance values.

Appendix C

DIFFERENTIAL SENSITIVITY

(see section 6)

Let us write

$$H = H_r + jH_i ,$$

where we have suppressed the dependency on s.

$$S_p^H = \frac{p}{H_r + jH_i}\left(\frac{\partial H_r}{\partial p} + j\,\frac{\partial H_i}{\partial p}\right)$$

$$= \frac{\left(H_r \frac{\partial H_r}{\partial p} + H_i \frac{\partial H_i}{\partial p}\right)p + j\left(H_r \frac{\partial H_i}{\partial p} - H_i \frac{\partial H_r}{\partial p}\right)p}{H_r^2 + H_i^2}$$

$$S_p^{|H|} = \frac{p}{\sqrt{H_r^2 + H_i^2}}\,\frac{1}{2}\left(H_r^2 + H_i^2\right)^{-\frac{1}{2}}\left(2H_r \frac{\partial H_r}{\partial p} + 2H_i \frac{\partial H_i}{\partial p}\right)$$

$$= \frac{p}{H_r^2 + H_i^2}\left(H_r \frac{\partial H_r}{\partial p} + H_i \frac{\partial H_i}{\partial p}\right)$$

$$= \text{real part of } S_p^H .$$

We now derive a formula for S_p^H. Consider for example an even order low-pass filter, so that the transfer function can be written as

$$H(s) = \prod_{k=1}^{n/2} F_k(s) .$$

Suppose that p refers to a resistance or capacitance value in the Lth section. Then

$$S_p^H = p\left(\prod_{\substack{k=1\\k\neq L}}^{n/2} F_k(s)\,\frac{\partial F_k(s)}{\partial p}\right)\Bigg/\prod_{k=1}^{n/2} F_k(s)$$

$$= \frac{p}{F_L(s)}\,\frac{\partial F_L(s)}{\partial p} .$$

038

For a low-pass filter $F_L(s)$ can be written as

$$F_L(s) = \frac{G\omega_0^2}{s^2 + \omega_0 s\big/Q + \omega_0^2}$$

so

$$\begin{aligned}\frac{\partial F_L(s)}{\partial p} = {} & G\left\{\left(s^2 + \omega_0 s/Q + \omega_0^2\right)2\omega_0 \frac{\partial \omega_0}{\partial p}\right. \\ & - \omega_0^2\left[\left(Q\frac{\partial \omega_0}{\partial p} - \omega_0 \frac{\partial Q}{\partial p}\right)\Big/Q^2\right]s \\ & \left. + 2\omega_0 \frac{\partial \omega_0}{\partial p}\right\}\Big/\left(s^2 + \omega_0 s\big/Q + \omega_0^2\right)^2 \\ & + \omega_0^2 \frac{\partial G}{\partial p}\Big/\left(s^2 + \omega_0 s\big/Q + \omega_0^2\right) \quad .\end{aligned}$$

Whence

$$\begin{aligned}S_p^H = {} & p\left\{\left(s^2 + \omega_0 s\big/Q + \omega_0^2\right)2\,\frac{\partial \omega_0}{\partial p}\Big/\omega_0\right. \\ & \left. - \left[\left(Q\frac{\partial \omega_0}{\partial p} - \omega_0 \frac{\partial Q}{\partial p}\right)\Big/Q^2\right]s + 2/\omega_0 \frac{\partial \omega_0}{\partial p}\right\}\Big/\left(s^2 + \omega_0 s/Q + \omega_0^2\right) + \frac{p}{G}\frac{\partial G}{\partial p} \quad .\end{aligned}$$

Similar formulae can be obtained for other types of filter.

Appendix D

DERIVATION OF TRANSMISSION MATRIX

(see section 7)

With non-ideal operational amplifiers the second order Ring-of-Three section is modelled as shown in Fig 21. Considering the nodes from 2 to 10 in turn we have the equations

$$\frac{V_1 - V_2}{R_6} = \frac{V_2 - V_0}{R_4} + \frac{V_2}{R_{in}} + \frac{V_2 - V_4}{Z_1}$$

$$V_3 = -A_0 V_2$$

$$\frac{V_2 - V_4}{Z_1} + \frac{V_3 - V_4}{R_{out}} = \frac{V_4 - V_5}{R_1}$$

$$\frac{V_4 - V_5}{R_1} = \frac{V_5 - V_7}{R_2} + \frac{V_5}{R_{in}}$$

$$V_6 = -A_0 V_5$$

$$\frac{V_6 - V_7}{R_{out}} + \frac{V_5 - V_7}{R_2} = \frac{V_7 - V_8}{R_3}$$

$$\frac{V_7 - V_8}{R_3} = \frac{V_8}{R_{in}} + \frac{V_8 - V_{10}}{Z_2}$$

$$V_9 = -A_0 V_8$$

$$\frac{V_9 - V_{10}}{R_{out}} + \frac{V_8 - V_{10}}{Z_2} + \frac{V_2 - V_{10}}{R_4} = -I_{out}$$

where R_{in}, R_{out}, A_0 are the input impedance, output impedance and open-loop gain of the amplifier, and comparing Figs 19 and 21 we see that

$$Z_1 = R_5/(1 + R_5 C_2 s)$$

$$Z_2 = 1/(C_1 s) \quad .$$

Manipulation of these equations gives

$$\frac{1}{R_6} V_1 - \left(\frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{R_{in}} + \frac{1}{Z_1}\right) V_2 + \frac{1}{Z_1} V_4 + \frac{1}{R_4} V_{10} = 0$$

$$\left(\frac{1}{Z_1} - \frac{A_0}{R_{out}}\right) V_2 - \left(\frac{1}{Z_1} + \frac{1}{R_{out}} + \frac{1}{R_1}\right) V_4 + \frac{1}{R_1} V_5 = 0$$

$$\frac{1}{R_1} V_4 - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}}\right) V_5 + \frac{1}{R_2} V_7 = 0$$

$$\left(\frac{1}{R_2} - \frac{A_0}{R_{out}}\right) V_5 - \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{out}}\right) V_7 + \frac{1}{R_3} V_8 = 0$$

$$\frac{1}{R_3} V_7 - \left(\frac{1}{R_3} + \frac{1}{Z_2} + \frac{1}{R_{in}}\right) V_8 + \frac{1}{Z_2} V_{10} = 0$$

$$\frac{1}{R_4} V_2 + \left(\frac{1}{Z_2} - \frac{A_0}{R_{out}}\right) V_8 - \left(\frac{1}{R_4} + \frac{1}{Z_2} + \frac{1}{R_{out}}\right) V_{10} = - I_{out}$$

Call these equations (7-1). We wish to determine the elements of the open circuit impedance matrix

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} ,$$

where

$$Z_{11} = \left.\frac{V_1}{I_{in}}\right|_{I_{out}=0} , \qquad Z_{12} = \left.\frac{V_1}{I_{out}}\right|_{I_{in}=0} ,$$

$$Z_{21} = \left.\frac{V_{10}}{I_{in}}\right|_{I_{out}=0} , \qquad Z_{22} = \left.\frac{V_{10}}{I_{out}}\right|_{I_{in}=0} .$$

(The positive inwards current convention is used.)

We have

$$I_{in} = (V_1 - V_2)/R_6 .$$

Define

$$u_1 = V_2/V_1$$

$$u_2 = V_4/V_1$$

$$u_3 = V_5/V_1$$

$$u_4 = V_7/V_1$$

$$u_5 = V_8/V_1$$

$$u_6 = V_{10}/V_1 \quad .$$

Consider first the situation when $I_{out} = 0$. We can re-write equations (7-1) in the form

$$M\underline{u} = \underline{b}$$

where M is a 6×6 matrix and

$$b_1 = -1/R_6 \;,\; b_2 = b_3 = b_4 = b_5 = b_6 = 0 \quad .$$

These equations can be solved for $\underline{u}$ and we shall then have

$$Z_{11} = V_1 R_6/(V_1 - V_2) = R_6/(1 - u_1)$$

$$Z_{21} = (V_{10}/V_1)/(V_1/I_{in}) = u_6 Z_{11} \quad .$$

Now consider the situation when $I_{in} = 0$. Equations (7-1) will still apply, except that we should replace the last equation by

$$V_1 = V_2 \quad \textit{ie} \quad u_2 = 1 \quad .$$

So we still have $M\underline{u} = \underline{b}$ with M being unchanged except for its last row and $\underline{b}$ being unchanged except that now $b_6 = 1$. From the last equation of (7-1)

$$I_{out}/V_1 = -\frac{1}{R_4} u_1 - \left(\frac{1}{Z_2} - \frac{A_0}{R_{out}}\right) u_5 + \left(\frac{1}{R_4} + \frac{1}{Z_2} + \frac{1}{R_{out}}\right) u_6 \quad ,$$

whence

$$Z_{12} = V_1/I_{out} \quad \text{and}$$

$$Z_{22} = u_6 Z_{12} \quad \text{can be found .}$$

038

The transmission matrix can now be found from the relations

$$a_{11} = Z_{11}/Z_{21} \qquad a_{12} = \Delta/Z_{21}$$

$$a_{21} = 1/Z_{21} \qquad a_{22} = Z_{22}/Z_{21}$$

where $\Delta = Z_{11}Z_{22} - Z_{12}Z_{21}$.

LIST OF SYMBOLS AND ABBREVIATIONS

a	negative of pole of transfer function of first order section
$a_{11}, a_{12}, a_{21}, a_{22}$	elements of matrix $\mathcal{A}$
A	coefficient of s in denominator of second order transfer function
A_0	operational amplifier open-loop gain
A_k	coefficient of s in denominator of transfer function of kth second order section
$\mathcal{A}$	transmission matrix
ACFA	name of program to calculate attenuation performance of active filter
ACFC	name of program to calculate component values of active filter
ACFD	name of program to calculate delay performance of active filter
B	constant coefficient in denominator second order transfer function
B_k	constant coefficient in denominator of transfer function of kth second order section
$B_n(x)$	nth order Bessel polynomial
C	coefficient of s^2 in numerator of second order transfer function
C_k	coefficient of s^2 in numerator of transfer function of kth second order section
C_1, C_2 etc	capacitance values
C_{max}	maximum capacitance value
C_{min}	minimum capacitance value
d	coefficient of s in numerator of first order transfer function
D	coefficient of s in numerator of second order transfer function
D_k	coefficient of s in numerator of transfer function of kth second order section
$D(\omega)$	delay at frequency ω
e	constant coefficient in numerator of first order transfer function
E	constant coefficient in numerator of second order transfer function
E_k	constant coefficient in numerator of transfer function of kth second order section
$F_k(s)$	transfer function of kth second order section
G	absolute value of gain of filter or filter section
$H(s)$	filter transfer function

LIST OF SYMBOLS AND ABBREVIATIONS (concluded)

I_{in}	input current
I_{out}	output current
n	filter order
p	arbitrary parameter
P	operational amplifier gain-bandwidth product
Q	quality factor
r	variable measuring the performance of a circuit
R_1, R_2 etc	resistance values
R_{max}	maximum resistance value
R_{min}	minimum resistance value
s	Laplace transform variable = $j\omega$
S_p^r	differential sensitivity of variable r with respect to change in parameter p
$T_n(x)$	nth order Papoulis polynomial
u_1, u_2, etc	voltage ratios
$U_n(x)$	nth order Halpern polynomial
V_1, V_2, etc	voltages
$Y_n^{\alpha}(x)$	nth order Ultraspherical polynomial
$Z_{11}, Z_{12}, Z_{21}, Z_{22}$	elements of Z matrix
Z	open circuit impedance matrix
Z_1, Z_2	impedances used in calculating non-ideal performance of Ring of Three circuit
α	parameter relating to Ultraspherical filters
γ	attenuation at $\omega = 1$
τ	time constant used in the definition of State Variable design equations
ω	arbitrary frequency in radians per second
ω_0	resonant frequency: $\omega_0^2 = B$

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```
← COMPILE ACFA
   . . .
← EXECUTE
   . . .
TYPE L, H OR B FOR LOW, HIGH OR BAND-PASS

← H

TYPE FILTER NAME

← BESS

TYPE ORDER OF FILTER

← 4

TYPE FREQUENCY OF PASS BAND EDGE

← 1.5&3

HOW MANY FREQUENCIES OF INTEREST?

← 3

TYPE FREQUENCIES OF INTEREST

← 1.2&3     1.0&3     0.8&3

ATTENUATION AT FREQ.    1.20000000&3 WITH ORDER 4  IS  4.9398197139&0

ATTENUATION AT FREQ.    1.00000000&3 WITH ORDER 4  IS  7.4218228870&0

ATTENUATION AT FREQ.    8.00000000&2 WITH ORDER 4  IS  1.1849284681&1
```

TR 79038

Fig 1 Example 1 of ACFA

Fig 2

```
← COMPILE ACFA
   . . .
← EXECUTE
   . . .
TYPE L, H OR B FOR LOW, HIGH OR BAND-PASS

← B

TYPE FILTER NAME

← CHEB

TYPE ORDER OF FILTER

← 0

TYPE ATTENUATION AT STOP BAND FREQUENCY

← 70.0

TYPE CENTRE FREQUENCY AND BANDWIDTH

← 500.0    50.0

TYPE ATTENUATION AT PASS BAND EDGE

← 0.1

HOW MANY FREQUENCIES OF INTEREST?

← 1

TYPE FREQUENCIES OF INTEREST

← 400.0

ATTENUATION AT FREQ.    4.00000000&2 WITH ORDER 1   IS   1.6781430083&0

BAND-PASS ORDER IS  +2

ATTENUATION AT FREQ.    4.00000000&2 WITH ORDER 2   IS   1.5722079393&1

BAND-PASS ORDER IS  +4

ATTENUATION AT FREQ.    4.00000000&2 WITH ORDER 3   IS   3.4579908261&1

BAND-PASS ORDER IS  +6

ATTENUATION AT FREQ.    4.00000000&2 WITH ORDER 4   IS   5.3553971786&1

BAND-PASS ORDER IS  +8

ATTENUATION AT FREQ.    4.00000000&2 WITH ORDER 5   IS   7.2529527521&1

BAND-PASS ORDER IS  +10
```

Fig 2 Example 2 of ACFA

TR 79038

```
← COMPILE ACFD
   . . .
← EXECUTE
   . . .
TYPE L, H OR B FOR LOW, HIGH OR BAND-PASS

← L

TYPE FILTER NAME

← CHEB

TYPE ORDER OF FILTER

← 5

TYPE FREQUENCY OF PASS BAND EDGE

← 1.0&3

TYPE ATTENUATION AT PASS BAND EDGE

← 0.28

HOW MANY FREQUENCIES OF INTEREST?

← 5

TYPE FREQUENCIES OF INTEREST

← 100.0    500.0    900.0    1.0&3    1.4&3

CHEBYCHEV FILTER

DELAY AT FREQ.    1.000000000&2 WITH ORDER 5  IS  6.1106389616&-4

DELAY AT FREQ.    5.000000000&2 WITH ORDER 5  IS  6.7223884753&-4

DELAY AT FREQ.    9.000000000&2 WITH ORDER 5  IS  9.8636728223&-4

DELAY AT FREQ.    1.000000000&3 WITH ORDER 5  IS  1.4414873339&-3

DELAY AT FREQ.    1.400000000&3 WITH ORDER 5  IS  2.6387288493&-4
```

Fig 3 Example of ACFD

Fig 4

```
← COMPILE ACFC
       . . .
← GETEBM PROGC
       . . .
← AS *TRO, LJRDAT

← AS *LPO, LJRRES

← EXECUTE
       . . .
← ACFTIDY LJRRES
```

Fig 4 Non-interactive run of ACFC

TR 79038

```
← LF LJRDAT
6    4    5    3
1.0
500.0    20.0
1.0&4
1.0&-11    1.0&-7
1.0&3      1.0&6
N
N
Y
Y
10.0
0
N
N
N
N
```

Fig 5   Data for non-interactive run of ACFC

```
++++++++++++++++++ +++ +++++++ +++++++++++ +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

&LISTING OF :RN46AUG.LJRKES(3/) PRODUCED ON 18SEP78 AT 15.53.21

&PRINTED ON REMOT. PRINTER 1A OF RAE 1906S BY G4 MK8.62 FOR JOB ':RN46AUG.R-R07' ON 26SEP78 AT 11.20.31

&DOCUMENT    LJRKES

BESSEL FILTER

NORMALISED ROOTS
SECTION        +1
+3.1629680R000& +0 +2.5725565713& +0
QUALITY +5.1031783602& -1   RES FREQ +1.6039191287& +0
SECTION        +2
+2.7657101950& +0 +2.6532694368& +0
QUALITY +6.1119654685& -1   RES FREQ +1.6891682678& +0
SECTION        +3
+1.8613150600& +0 +3.6279110836& +0
QUALITY +1.0233139553& +0   RES FREQ +1.9067076123& +0
DENORMALISED ROOTS
SECTION        +1
+1.9621126963& +2 +9.7436682155& +6
QUALITY +1.5906793750& +1   RES FREQ +3.1214849380& +3
SECTION        +2
+1.7027682652& +2 +9.6933210102& +6
QUALITY +1.8005002055& +1   RES FREQ +3.0811233357& +3
SECTION        +3
+1.1306611000& +2 +9.2346648178& +6
QUALITY +2.6877000808& +1   RES FREQ +3.0388920372& +3
SECTION        +4
+1.9874732607& +2 +9.9971682956& +6
QUALITY +1.5908792009& +1   RES FREQ +3.1618298973& +3
SECTION        +5
+1.7702399355& +2 +1.0260802403& +7
QUALITY +1.8064997552& +1   RES FREQ +3.2032687263& +3
SECTION        +6
+1.2083538505& +2 +1.0547971519& +7
QUALITY +2.6877591183& +1   RES FREQ +3.2477640790& +3

RING OF THREE BANDPASS

SECTION    1
CAPACITANCES 1.000000& -9 1.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 3.203603& +5 3.203603& +5 5.096547& +6 4.993600& +6
SECTION    1
CAPACITANCES 2.200000& -9 2.200000& -9
RESISTANCES 1.000000& +4 1.000000& +4 1.456183& +5 1.456183& +5 2.316612& +6 2.269727& +6
SECTION    1
CAPACITANCES 5.000000& -9 5.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 6.407207& +4 6.407207& +4 1.019309& +6 9.986799& +5
SECTION    1
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.203603& +4 3.203603& +4 5.096547& +5 4.993600& +5
SECTION    2
CAPACITANCES 1.000000& -9 1.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 3.245570& +5 3.245570& +5 5.872859& +6 4.803503& +6
SECTION    2
CAPACITANCES 2.200000& -9 2.200000& -9
RESISTANCES 1.000000& +4 1.000000& +4 1.475259& +5 1.475259& +5 2.669481& +6 2.183610& +6
SECTION    2
CAPACITANCES 5.000000& -9 5.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 6.491139& +4 6.491139& +4 1.174572& +6 9.607006& +5
```

PRACTICABLE

Fig 6   Results of non-interactive run of ACFC

```
SECTION    2
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.245570& +4 3.245570& +4 5.872859& +5 4.803503& +5
SECTION    3
CAPACITANCES 1.000000& -9 1.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 3.290673& +5 3.290673& +5 8.844540& +6 4.319132& +6
SECTION    3
CAPACITANCES 2.200000& -9 2.200000& -9
RESISTANCES 1.000000& +4 1.000000& +4 1.495760& +5 1.495760& +5 4.020245& +6 1.963242& +6
SECTION    3
CAPACITANCES 5.000000& -9 5.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 6.581346& +4 6.581346& +4 1.768906& +6 8.638263& +5
SECTION    3
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.290673& +4 3.290673& +4 8.844540& +5 4.319132& +5
SECTION    4
CAPACITANCES 1.000000& -9 1.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 3.162725& +5 3.162725& +5 5.031514& +6 4.929683& +6
SECTION    4
CAPACITANCES 2.200000& -9 2.200000& -9
RESISTANCES 1.000000& +4 1.000000& +4 1.437602&.+5 1.437602& +5 2.287052& +6 2.240765& +6
SECTION    4
CAPACITANCES 5.000000& -9 5.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 6.325451& +4 6.325451& +4 1.006303& +6 9.859367& +5
SECTION    4
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.162725& +4 3.162725& +4 5.031514& +5 4.929683& +5
SECTION    5
CAPACITANCES 1.000000& -9 1.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 3.121831& +5 3.121831& +5 5.648952& +6 4.620366& +6
SECTION    5
CAPACITANCES 2.200000& -9 2.200000& -9
RESISTANCES 1.000000& +4 1.000000& +4 1.419014& +5 1.419014& +5 2.567705& +6 2.100166& +6
SECTION    5
CAPACITANCES 5.000000& -9 5.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 6.243661& +4 6.243661& +4 1.129790& +6 9.240732& +5
SECTION    5
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.121831& +4 3.121831& +4 5.648952& +5 4.620366& +5
SECTION    6
CAPACITANCES 1.000000& -9 1.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 3.079041& +5 3.079041& +5 8.275722& +6 4.041357& +6
SECTION    6
CAPACITANCES 2.200000& -9 2.200000& -9
RESISTANCES 1.000000& +4 1.000000& +4 1.399564& +5 1.399564& +5 3.761692& +6 1.836980& +6
SECTION    6
CAPACITANCES 5.000000& -9 5.000000& -9
RESISTANCES 1.000000& +4 1.000000& +4 6.158083& +4 6.158083& +4 1.655144& +6 8.082714& +5
SECTION    6
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.079041& +4 3.079041& +4 8.275722& +5 4.041357& +5

COMPONENT VALUES
----------------

SECTION    1
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.263603& +4 3.263603& +4 5.026547& +5 4.997400& +5
SECTION    2
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.245570& +4 3.245570& +4 5.872859& +5 4.803503& +5
SECTION    3
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.290673& +4 3.290673& +4 8.844540& +5 4.319132& +5
SECTION    4
CAPACITANCES 1.000000& -8 1.000000& -8
RESISTANCES 1.000000& +4 1.000000& +4 3.162725& +4 3.162725& +4 5.031514& +5 4.929683& +5
```

Fig 6 contd

```
SECTION 5
CAPACITANCES 1.[illegible] +0 1.[illegible] +0
RESISTANCES 1.[illegible] +6 1.[illegible] +6 3.12[illegible]31E +6 3.121831E +6 5.668052E +5 4.620366E +5
SECTION 6
CAPACITANCES 1.[illegible] +0 1.[illegible] +0
RESISTANCES 1.[illegible] +6 1.[illegible] +6 3.[illegible]041E +6 3.370041E +6 8.275722E +5 4.041357E +5
OVGAIN = 1


DIFFERENTIAL SENSITIVITIES AT CENTRE FREQUENCY
ELEMENT     SENSITIVITY          TOLERANCE(%)
SECTION 1
C 1     -3.1[illegible]E +0         3.2E +0
C 2     -3.1[illegible]E +0         3.2E +0
R 1     -3.1[illegible]E +0         3.2E +0
R 2      3.1[illegible]E +0         3.2E +0
R 3     -3.1[illegible]E +0         3.2E +0
R 4     -3.1[illegible]E +0         3.2E +0
R 5      9.5[illegible]E -1         1.0E +1
R 6     -1.[illegible]E +0          1.0E +1
SECTION 2
C 1     -8.3512[illegible]07E +0    1.2E +0
C 2     -8.6[illegible]E +0         1.2E +0
R 1     -8.3[illegible]07E +0       1.2E +0
R 2      8.3512[illegible]07E +0    1.2E +0
R 3     -8.3512[illegible]07E +0    1.2E +0
R 4     -8.3512[illegible]07E +0    1.2E +0
R 5      6.6[illegible]53E -1       1.5E +1
R 6     -1.00000000E +0     1.0E +1
SECTION 3
C 1     -1.10794[illegible]70E +1   9.0E -1
C 2     -1.1[illegible]410[illegible]2E +1  8.4E -1
R 1     -1.10794770E +1     9.0E -1
R 2      1.10794770E +1     9.0E -1
R 3     -1.10794770E +1     9.0E -1
R 4     -1.10794770E +1     9.0E -1
R 5      2.5[illegible]59E -1       4.2E +1
R 6     -1.00000000E +0     1.0E +1
SECTION 4
C 1      3.14009[illegible]10E +0   3.2E +0
C 2      3.10002[illegible]45E +0   3.2E +0
R 1      3.14009710E +0     3.2E +0
R 2     -3.14009710E +0     3.2E +0
R 3      3.14009710E +0     3.2E +0
R 4      3.14009710E +0     3.2E +0
R 5      9.5[illegible]2351E -1     1.0E +1
R 6     -1.00000000E +0     1.0E +1
SECTION 5
C 1      8.63222425E +0     1.2E +0
C 2      8.35121[illegible]29E +0   1.2E +0
R 1      8.63222425E +0     1.2E +0
R 2     -8.63222425E +0     1.2E +0
R 3      8.63222425E +0     1.2E +0
R 4      8.63222425E +0     1.2E +0
R 5      6.6[illegible]580[illegible]40E -1  1.5E +1
R 6     -1.00000000E +0     1.0E +1
SECTION 6
C 1      1.1[illegible]410302E +1   8.4E -1
C 2      1.10794[illegible]51E +1   9.0E -1
R 1      1.1[illegible]410[illegible]02E +1  8.4E -1
R 2     -1.1[illegible]410[illegible]02E +1  8.4E -1
R 3      1.1[illegible]02E +1       8.4E -1
R 4      1.1[illegible]02E +1       8.4E -1
R 5      2.5[illegible]474[illegible]90E -1  4.2E +1
R 6     -1.[illegible]E +0          1.0E +1
```


Fig 6 contd

DIFFERENTIAL SENSITIVITIES AT CUTOFF FREQUENCY

ELEMENT	SENSITIVITY	TOLERANCE(%)
SECTION 1		
C 1	5.87108350E +0	1.7E +0
C 2	5.71350055E +0	1.8E +0
R 1	5.87108350E +0	1.7E +0
R 2	-5.87108350E +0	1.7E +0
R 3	5.87108350E +0	1.7E +0
R 4	5.87108350E +0	1.7E +0
R 5	8.62714585E -1	1.2E +1
R 6	-1.00000000E +0	1.0E +1
SECTION 2		
C 1	3.66759972E -1	2.7E +1
C 2	3.63546888E -1	2.7E +1
R 1	3.66759972E -1	2.7E +1
R 2	-3.66759972E -1	2.7E +1
R 3	3.66759972E -1	2.7E +1
R 4	3.66759972E -1	2.7E +1
R 5	9.95566087E -1	1.0E +1
R 6	-1.00000000E +0	1.0E +1
SECTION 3		
C 1	-1.25300185E +1	8.0E -1
C 2	-1.22662410E +1	7.8E -1
R 1	-1.25300185E +1	8.0E -1
R 2	1.25300185E +1	8.0E -1
R 3	-1.25300185E +1	8.0E -1
R 4	-1.25300185E +1	8.0E -1
R 5	6.63617155E -1	1.5E +1
R 6	-1.00000000E +0	1.0E +1
SECTION 4		
C 1	8.04593254E +0	1.2E +0
C 2	7.67112645E +0	1.3E +0
R 1	8.04593254E +0	1.2E +0
R 2	-8.04593254E +0	1.2E +0
R 3	8.04593254E +0	1.2E +0
R 4	8.04593254E +0	1.2E +0
R 5	5.85494295E -1	1.7E +1
R 6	-1.00000000E +0	1.0E +1
SECTION 5		
C 1	8.84520943E +0	1.1E +0
C 2	8.17440060E +0	1.2E +0
R 1	8.84520943E +0	1.1E +0
R 2	-8.84520943E +0	1.1E +0
R 3	8.84520943E +0	1.1E +0
R 4	8.84520943E +0	1.1E +0
R 5	3.20191722E -1	3.0E +1
R 6	-1.00000000E +0	1.0E +1
SECTION 6		
C 1	8.82455154E +0	1.1E +0
C 2	7.93326252E +0	1.5E +0
R 1	8.82455154E +0	1.1E +0
R 2	-8.82455154E +0	1.1E +0
R 3	8.82455154E +0	1.1E +0
R 4	8.82455154E +0	1.1E +0
R 5	1.08751172E -1	9.2E +1
R 6	-1.00000000E +0	1.0E +1

Fig 6 concld

Fig 7

```
TYPE NUMBERS FOR FILTER ORDER, FILTER TYPE, CIRCUIT TYPE AND LOW, HIGH
OR BAND-PASS
← 4    1    5    1
TYPE OVERALL GAIN
← 1.0
TYPE FREQUENCY OF PASS BAND EDGE
← 200.0
TYPE ARBITRARY VALUE FOR RESISTANCE
← 1.0&4
TYPE CMIN AND CMAX
← 1.0&-11    1.0&-7
TYPE RMIN AND RMAX
← 1.0&3    1.0&6
BUTTERWORTH FILTER
NORMALISED ROOTS
SECTION  +1
+1.8477590650&0      +1.00000000&0
QUALITY  +5.4119610015&-1
                                    RES FREQ. 1.00000000&0
SECTION   +2
+7.6536686473&-1    +1.00000000&0
QUALITY  +1.3065629649&0
                                    RES FREQ. 1.00000000&0
DENORMALISED ROOTS
SECTION   +1
+2.3219625217&3    +1.5791367042&6
QUALITY +5.4119610015&-1
                                    RES FREQ. 1.2566370614&3
```

Fig 7 Interactive run of ACFC

TR 79038/1

```
SECTION    +2

+9.6178836782&2    +1.5791367042&6

QUALITY 1.3065629649&0

                                        RES FREQ. 1.2566370614&3

RELATIVE GAINS TO BE SPECIFIED?

← NO

RING OF THREE LOW PASS

SECTION 1

CAPACITANCES     1.000000&-9     1.000000&-9

RESISTANCES      1.000000&4      1.000000&4

    7.957747&5     7.957747&5     4.306702&5

    7.957747&5

SECTION 2

CAPACITANCES     1.000000&-9     1.000000&-9

RESISTANCES      1.000000&4      1.000000&4

    7.957747&5     7.957747&5

    1.039730&6     7.957747&5

SECTION 2

CAPACITANCES     2.200000&-9     2.200000&-9

RESISTANCES      1.000000&4      1.000000&4

    3.617158&5     3.617158&5

    4.726044&4     3.617158&5

COMPONENT VALUES

SECTION 1

CAPACITANCES     1.000000&-9     1.000000&-9

RESISTANCES      1.000000&4      1.000000&4

    7.957747&5     7.957747&5

    4.306702&5     7.957747&5
```

```
SECTION 2

CAPACITANCES     2.2000000&-9     2.2000000&-9

RESISTANCES      1.0000000&4      1.0000000&4

    3.617158&5     3.617158&5     4.726044&5

    3.617158&5

OKGAIN  =  T

COMPONENT VALUES TO BE ADJUSTED?

← NO

SENSITIVITY ANALYSIS REQUIRED?

← Y

ARE DIFFERENTIAL SENSITIVITIES REQUIRED?

← N

TYPE NUMBER OF LARGE CHANGES

← 8

TYPE TRIPLES SPECIFYING CHANGES

← C1     1     2.0
  C2     1     2.0
  R1     1     5.0
  R2     1     5.0
  R3     1     5.0
  R4     1     5.0
  R5     1     5.0
  R6     1     5.0

PERCENTAGE CHANGE IN GAIN AT ZERO FREQUENCY -4.7619047626&0

PERCENTAGE CHANGE IN GAIN AT CUT-OFF FREQUENCY -4.7619047601&0

IMPEDANCES AND GAINS REQUIRED?

← Y

SECTIONS ORDERED WITH INCREASING QUALITY?

← Y

ORDER OF SECTIONS

1     2
```


Fig 7 contd

TYPE OP AMP INPUT IMPEDANCE, OUTPUT IMPEDANCE AND OPEN LOOP GAIN

← 1.0&6   300.0   1.0&5

TYPE NUMBER OF EXTRA FREQUENCIES

← 0

IMPEDANCES AND GAIN AT FREQUENCY 0.000000&0 HZ

INPUT AND OUTPUT IMPEDANCES OF SECTION 1

7.9577471562&5   9.9545550210&-3

INPUT AND OUTPUT IMPEDANCES OF SECTION 2

3.6171577980&5   3.1267580977&-3

MINIMUM VOLTAGE (AT END OF SECTION 2) IS 1.00000&0

MAXIMUM VOLTAGE (AT END OF SECTION 0) IS 1.00004&0

VOLTAGE SPREAD IS 1.00004&0

TRANSMISSION A-MATRIX FOR COMPLETE CIRCUIT

1.00004365&0 ? 0.00000000&0	3.12693670&-3 ? 0.00000000&0
1.25669192&-6 ? 0.00000000&0	3.92942455&-9 ? 0.00000000&0

IMPEDANCES AND GAIN OF COMPLETE CIRCUIT AT ZERO FREQUENCY

	MODULUS	PHASE (DEGREES)
INPUT	7.9577471562&5	0.0000000000&0
OUTPUT	3.1268002007&-3	0.0000000000&0
GAIN	9.9995634692&-1	0.0000000000&0

(= -3.7917413752&-4 DB)

IMPEDANCES AND GAIN AT FREQUENCY 2.000000&2 HZ

INPUT AND OUTPUT IMPEDANCES OF SECTION 1

7.9577915423&5   7.0112662168&-3

TR 79038/4

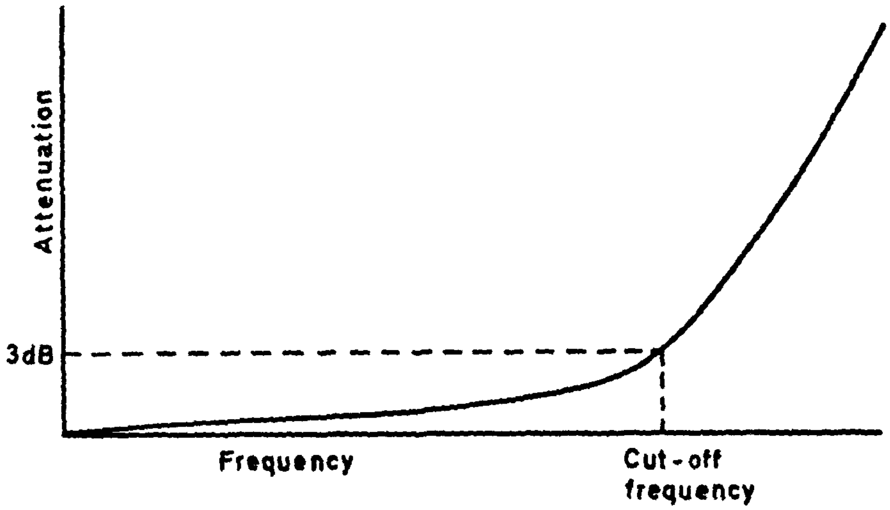
Fig 7 contd

```
INPUT AND OUTPUT IMPEDANCES OF SECTION 2

3.617206503585    8.3388153638-3

MINIMUM VOLTAGE (AT END OF SECTION 1) IS  7.65436&-1

MAXIMUM VOLTAGE (AT END OF SECTION 0) IS  1.41444&0

VOLTAGE SPREAD IS  1.84788&0

TRANSMISSION A-MATRIX FOR COMPLETE CIRCUIT

-1.41443559&0 ? -6.263163301&-5          -1.17922930&-2 ? 2.34149718&-4

-1.77742227&-6 ? -7.870138825&-11        -1.481854498&-8 ? 2.942396018&-10

IMPEDANCES AND GAIN OF COMPLETE CIRCUIT AT CUT-OFF FREQUENCY

       MODULUS                PHASE (DEGREES)

INPUT     7.9577915425&5      1.0979566933&-7

OUTPUT    8.3387447760&-3    -1.1400620578&0

GAIN      7.0699578220&-1     1.7999746293&2

                                          (= -3.0116635423&0 DB)

FURTHER ANALYSIS OF THIS CIRCUIT REQUIRED?

← YES

SENSITIVITY ANALYSIS REQUIRED?

← Y

ARE DIFFERENTIAL SENSITIVITIES REQUIRED?

← N

TYPE NUMBER OF LARGE CHANGES

← 8
```

```
TYPE TRIPLES SPECIFYING CHANGES

← C1    2    2.0
← C2    2    2.0
← R1    2    5.0
← R2    2    5.0
← R3    2    5.0
← R4    2    5.0
← R5    2    5.0
← R6    2    2.0

PERCENTAGE CHANGE IN GAIN AT ZERO FREQUENCY

             -1.9607843148&0

PERCENTAGE CHANGE IN GAIN AT CUT-OFF FREQUENCY

             -1.9607843136&0

IMPEDANCES AND GAINS REQUIRED?

← N

FURTHER ANALYSIS OF THIS CIRCUIT REQUIRED?

← N

COMPONENTS OR RELATIVE GAINS TO BE ADJUSTED?

← N

IS THERE MORE DATA?

← N
```


Fig 7 concld

Fig 8



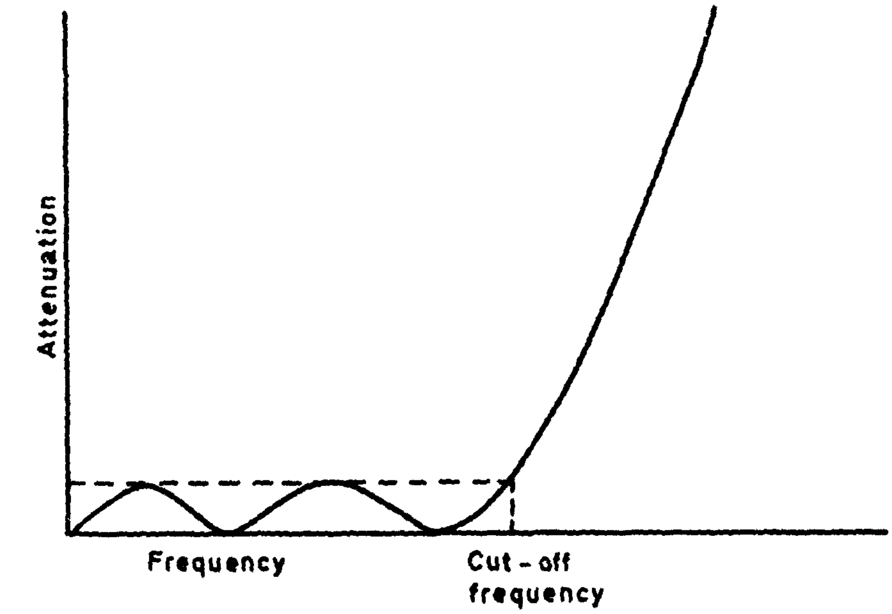
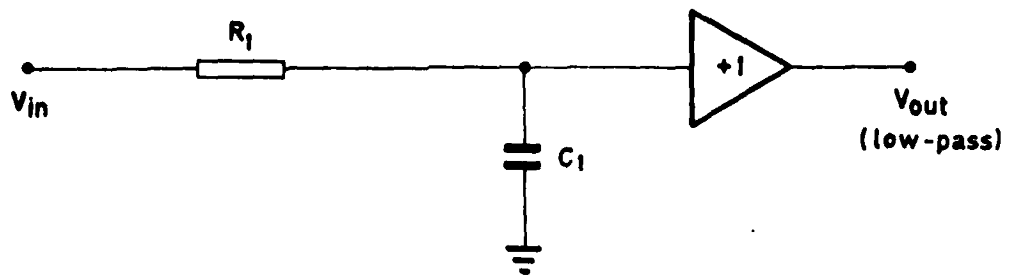


Fig 8 Definition of cut-off frequency

TR 79038



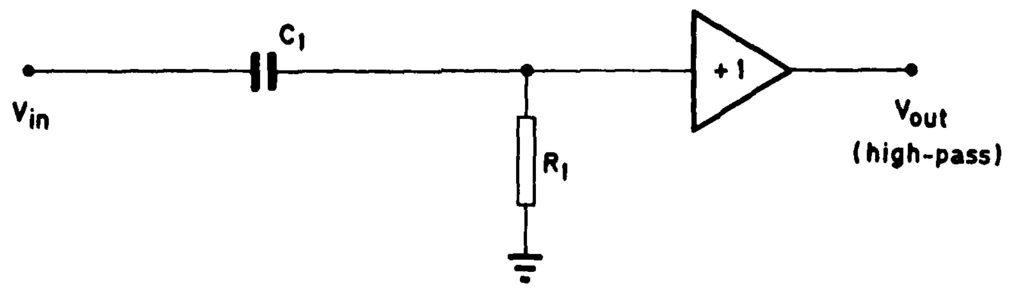


Fig 9 First order section

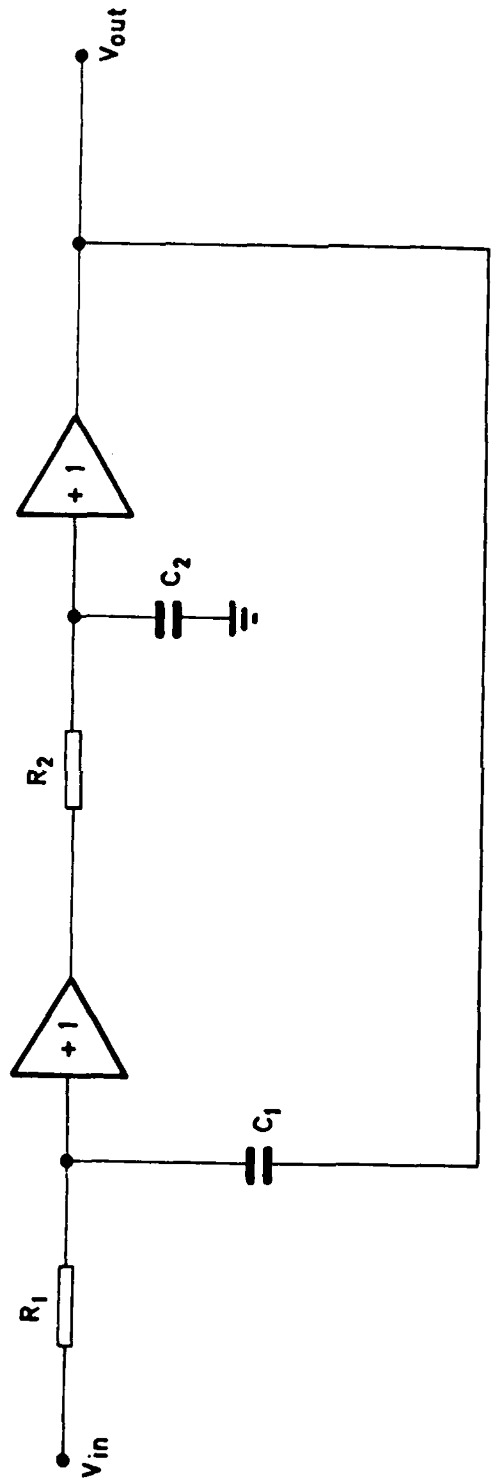


Fig 10 Unity gain — low-pass

TR 79038

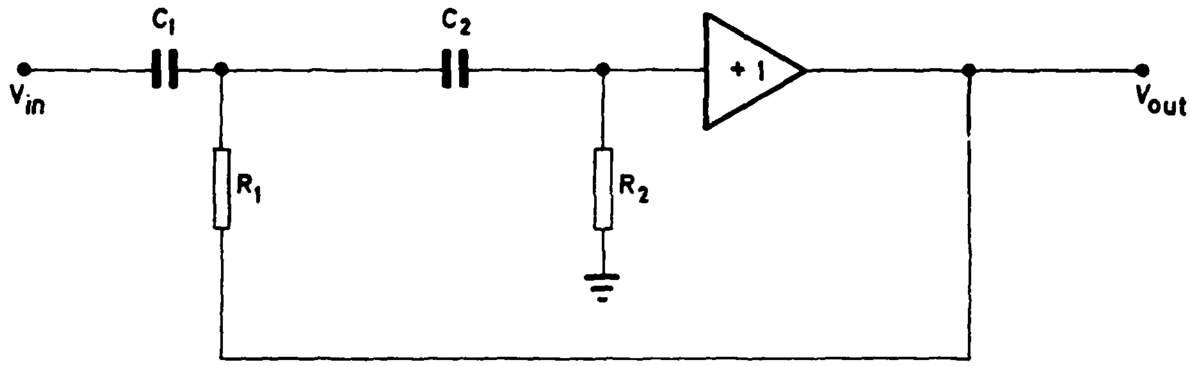


Fig 11 Unity gain – high-pass

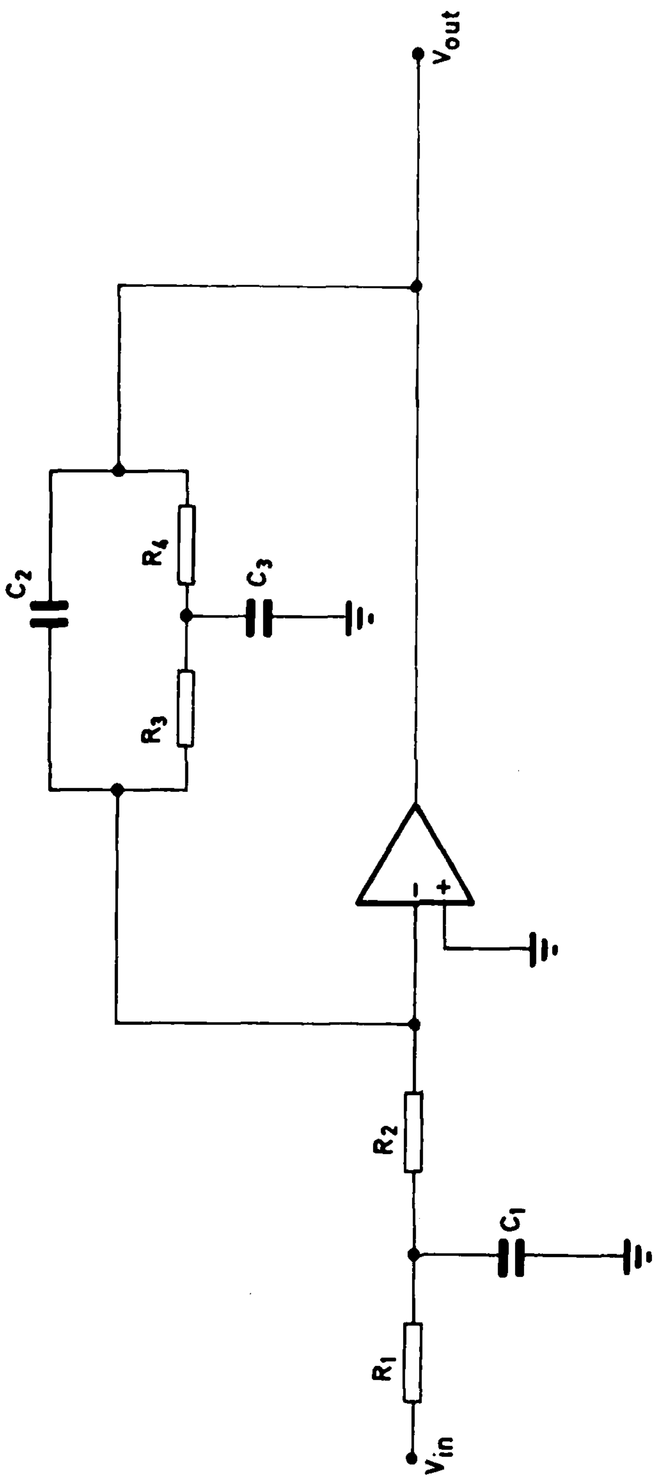


Fig 12 Single feedback — low-pass

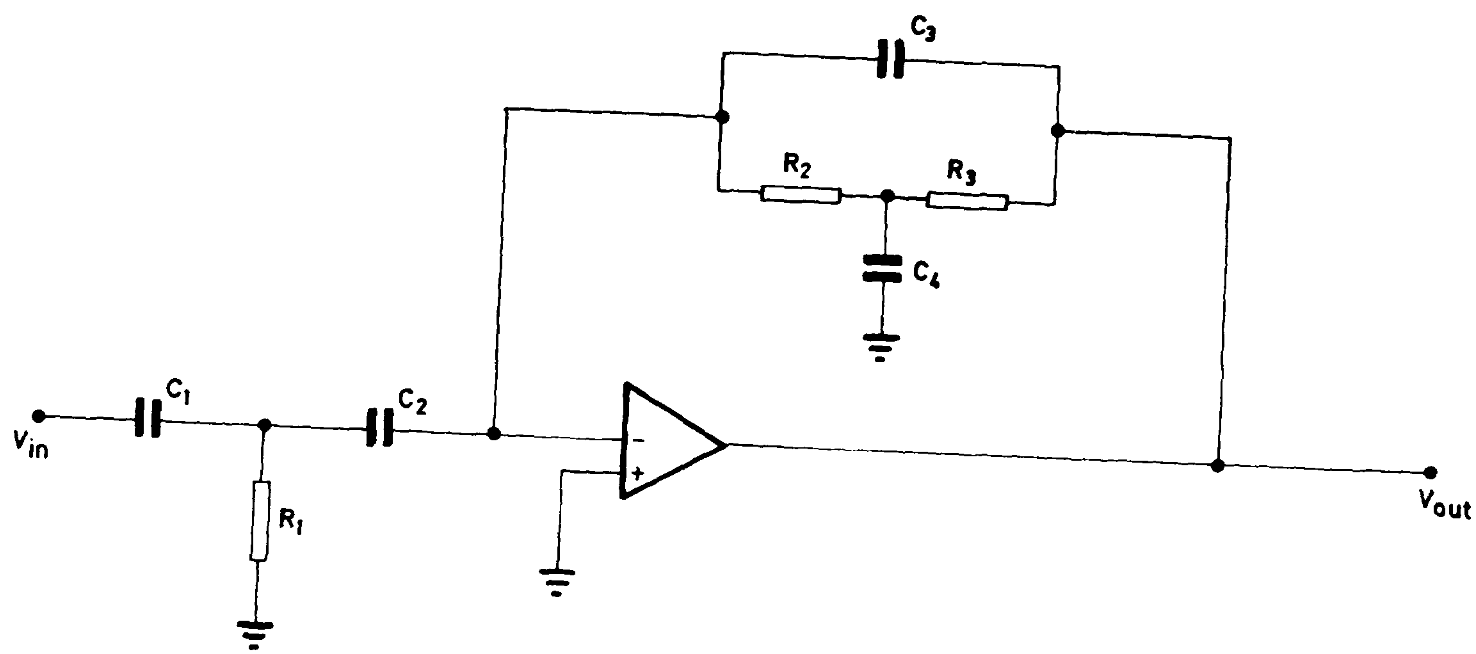


Fig 13 Single feedback – high-pass

Fig 14

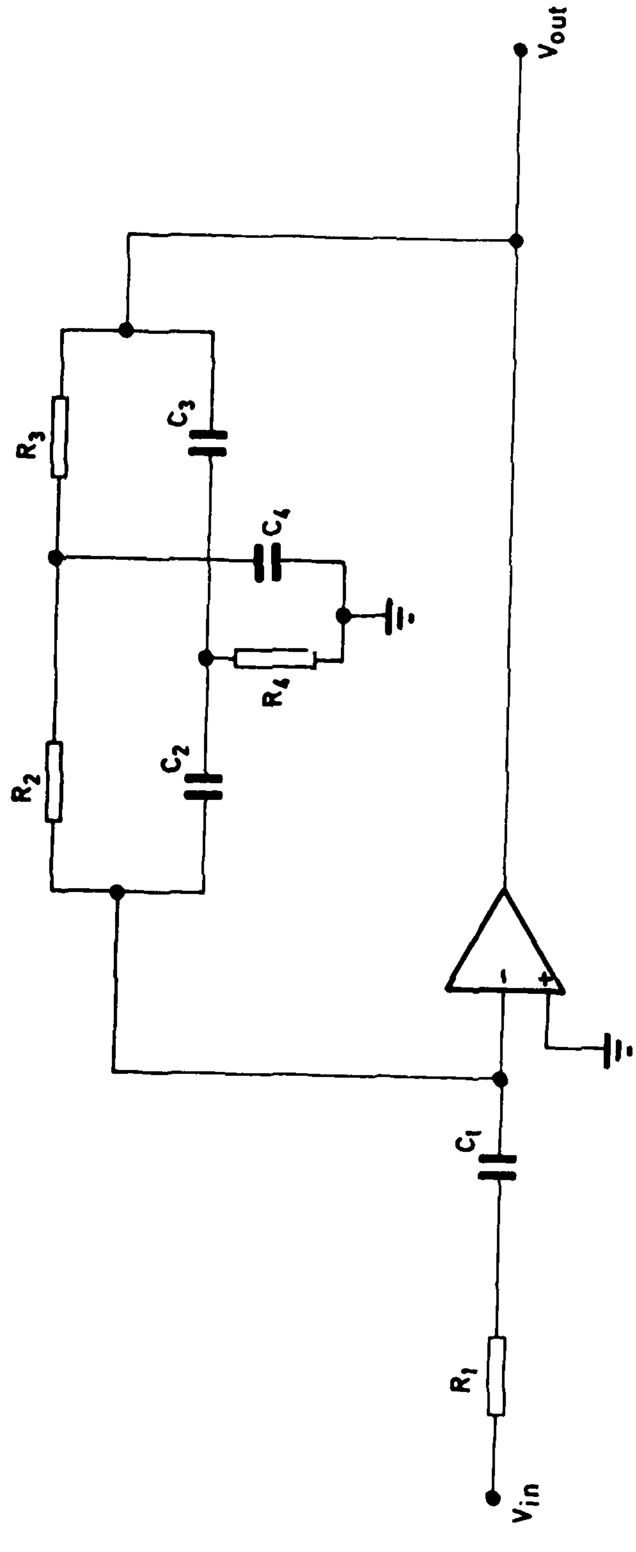


Fig 14 *Single feedback — band-pass*

TR 79038

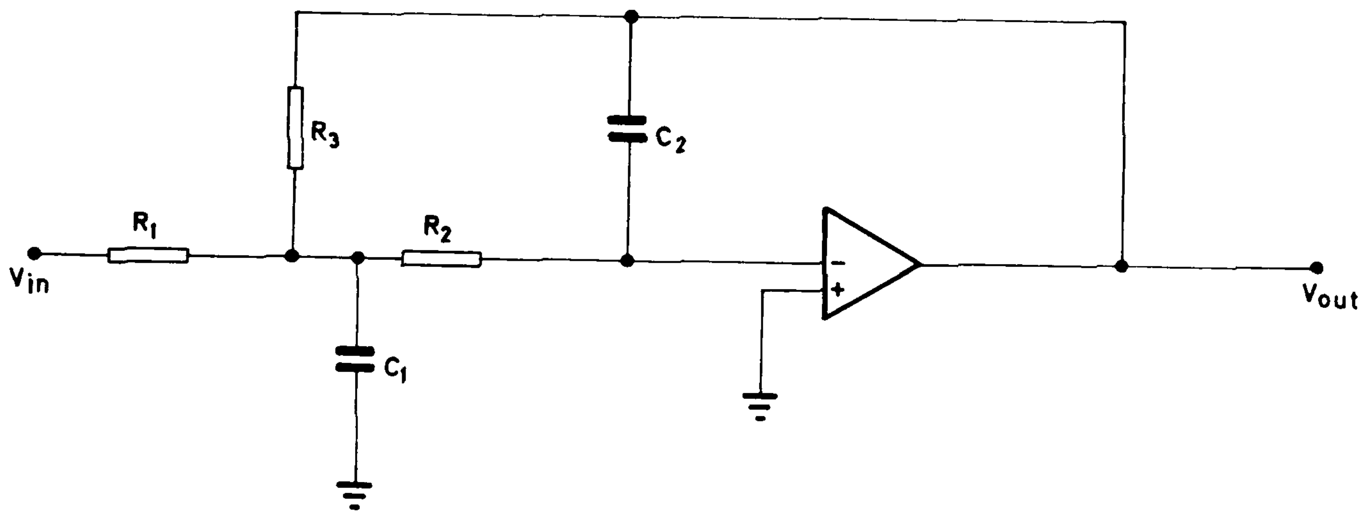


Fig 15 Multiple feedback – low-pass

Fig 16

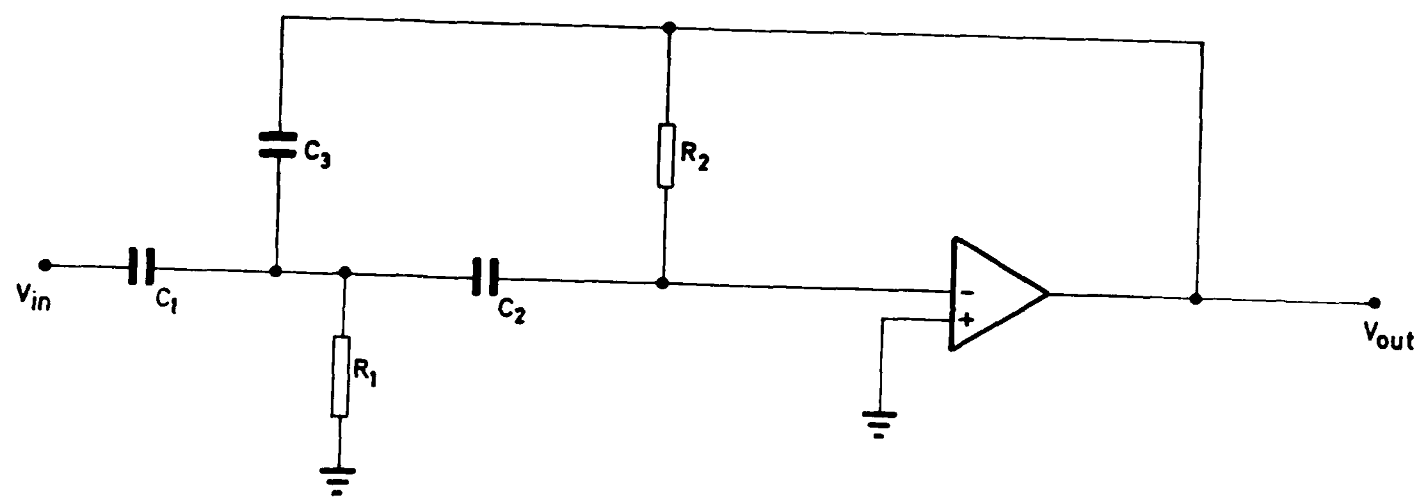


Fig 16 **Multiple feedback** – *high-pass*

TR 79038

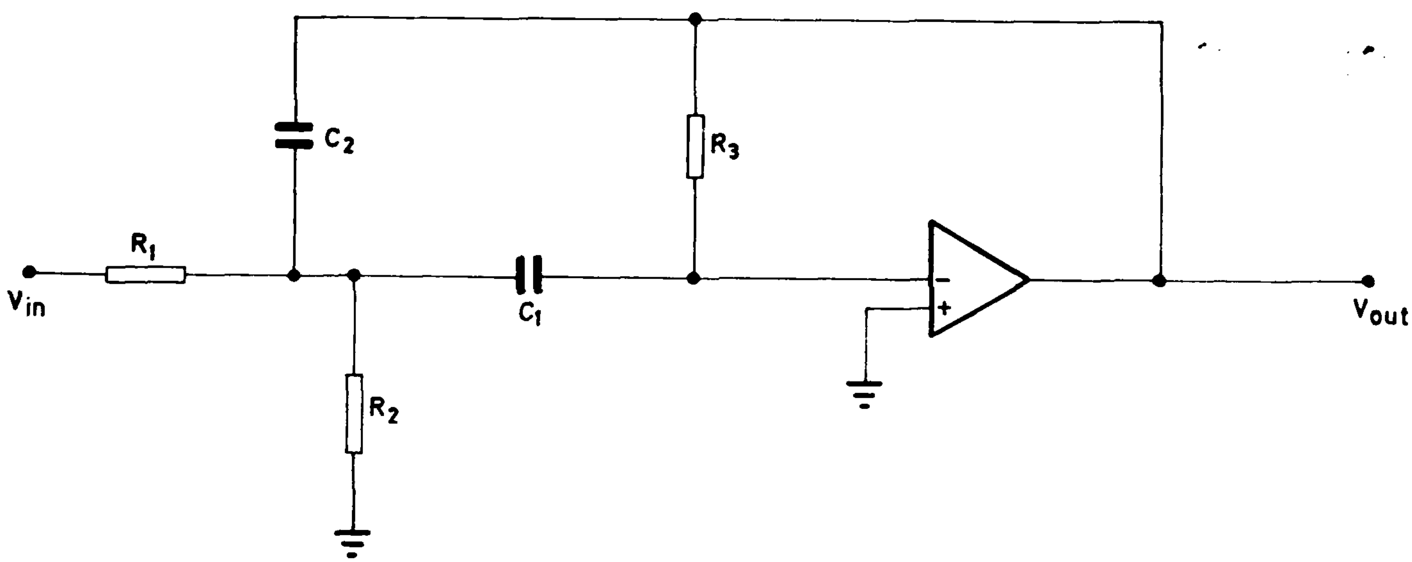


Fig 17 Multiple feedback – band-pass

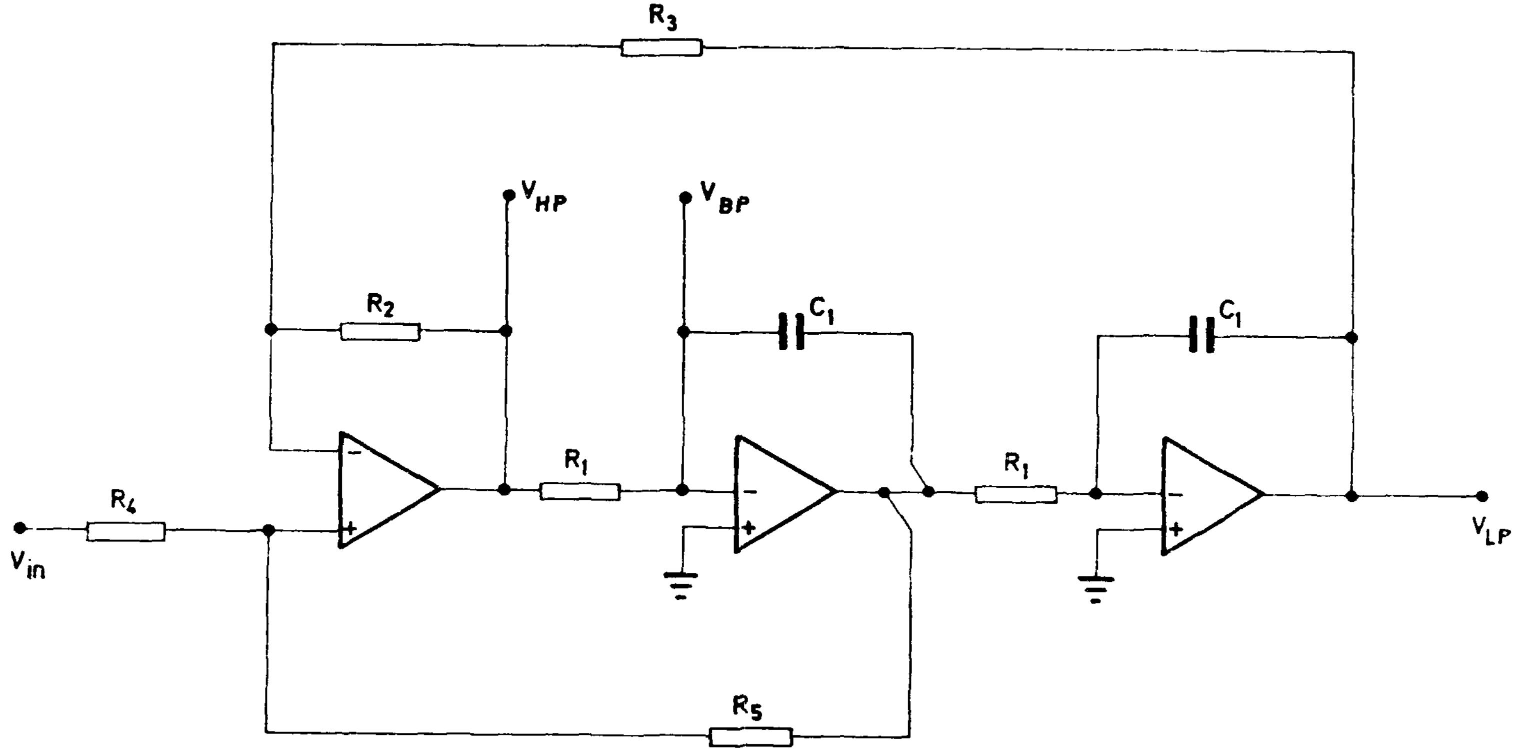


Fig 18 State variable circuit

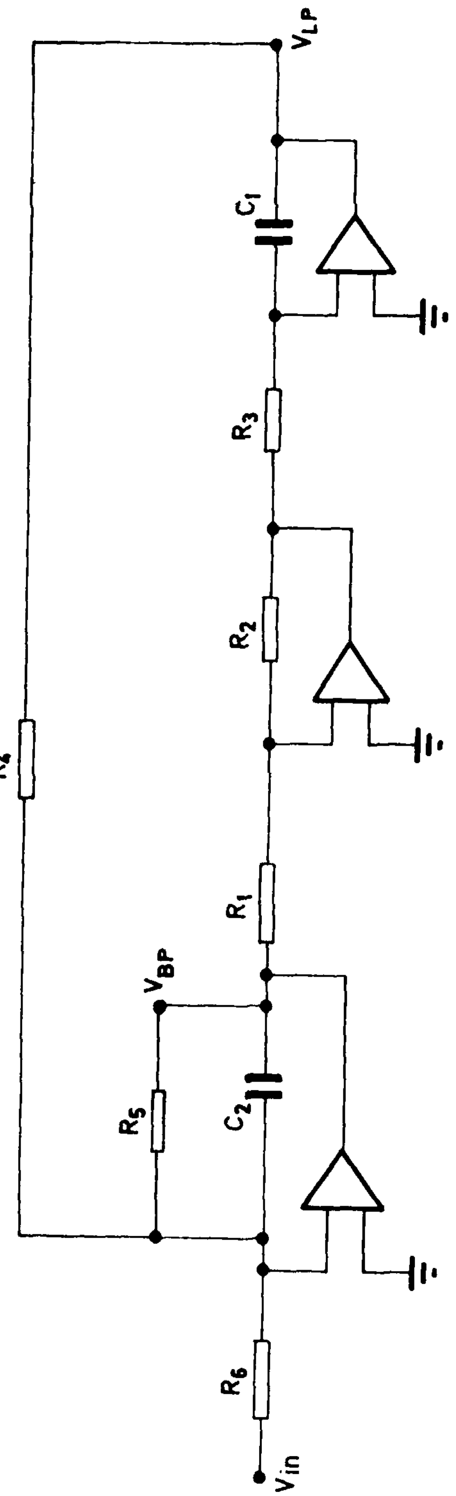


Fig 19 Ring-of-three circuit

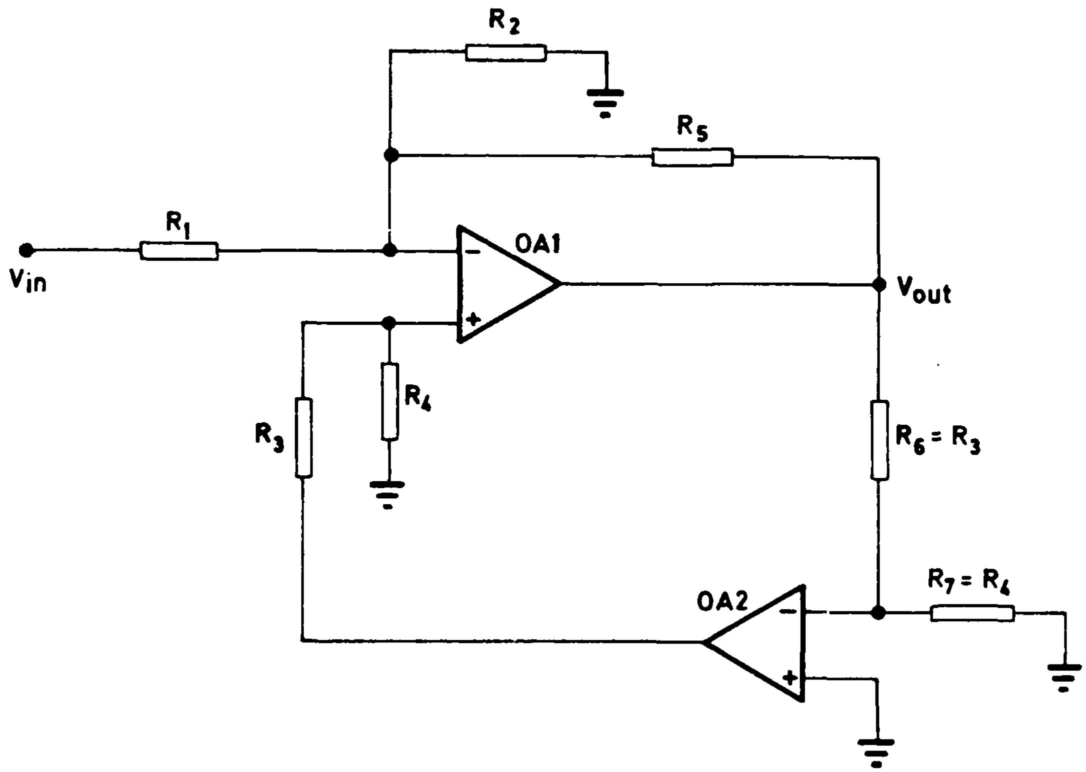
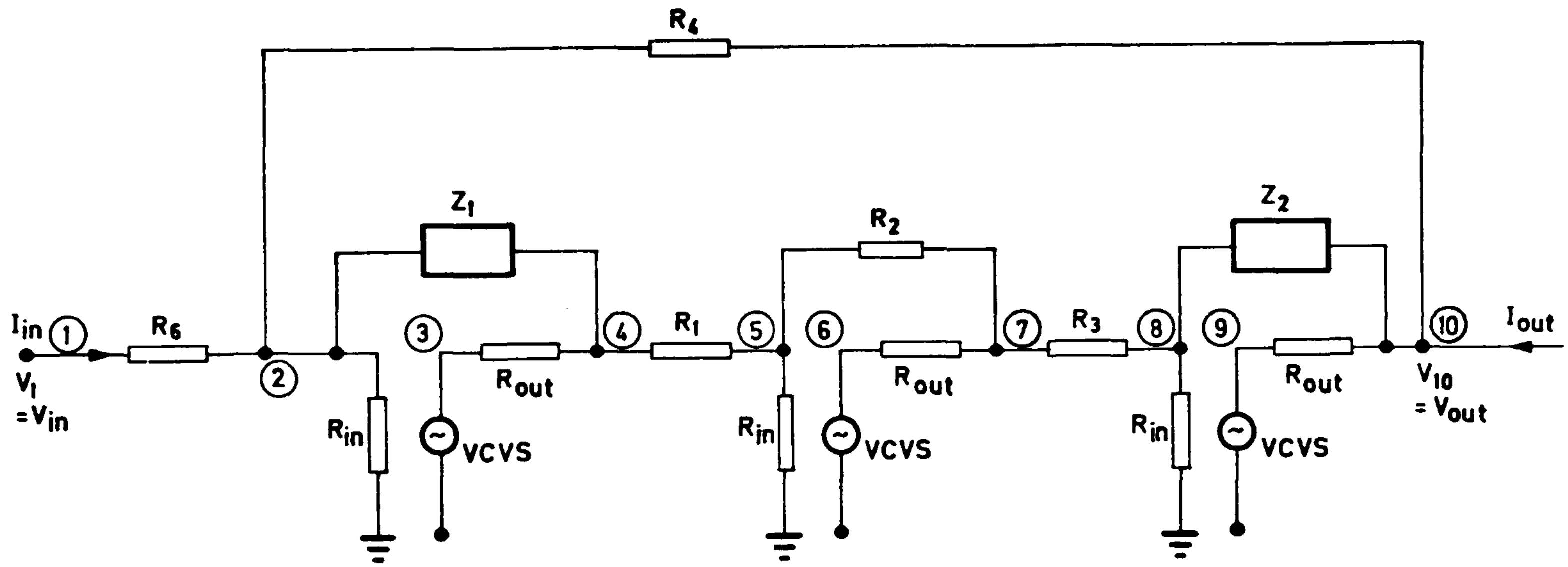


Fig 20 Mitra-Aatre band-pass

TR 79038



VCVS = voltage controlled voltage source

Fig 21 Ring-of-three low-pass model

Fig 22

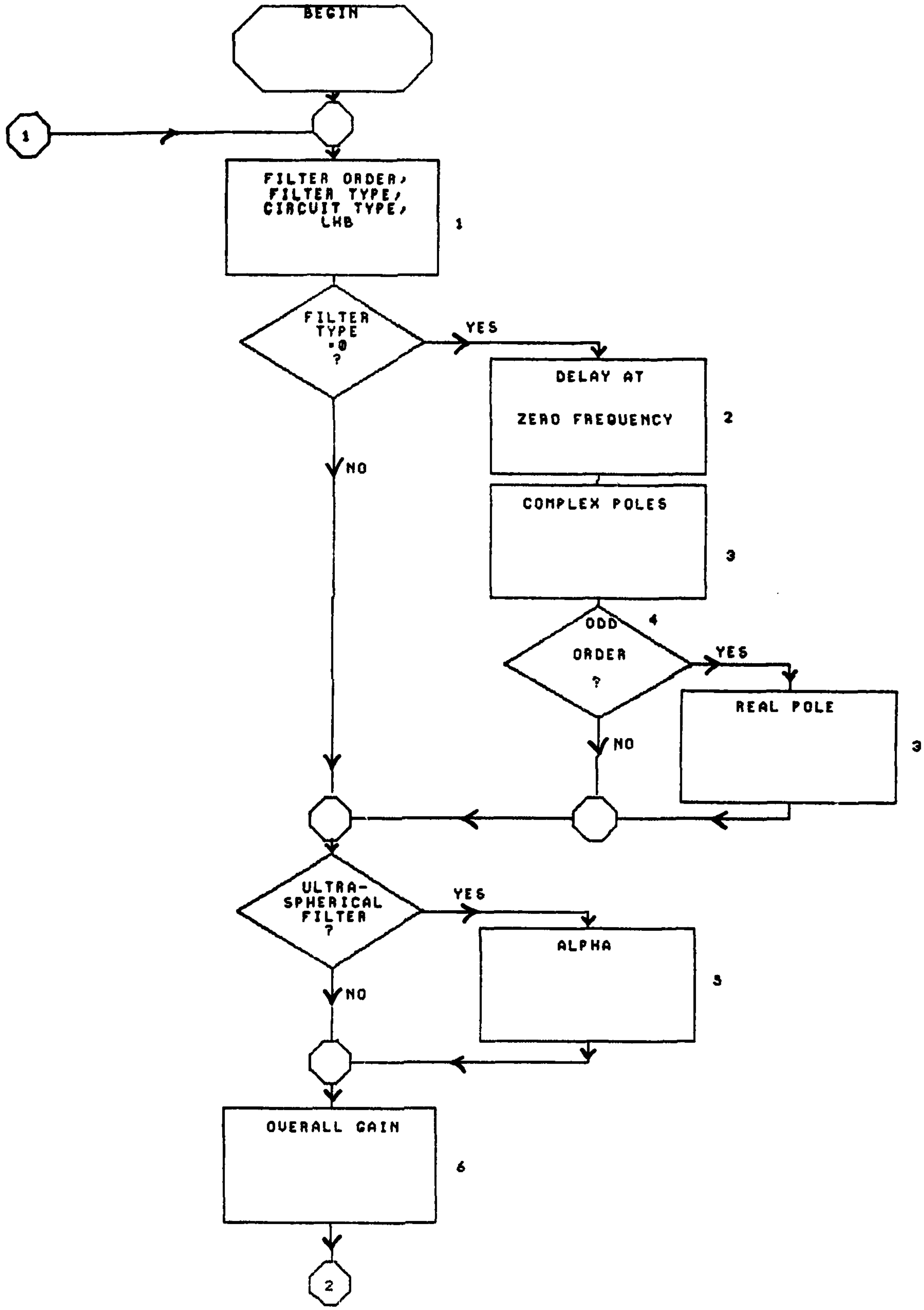


Fig 22 Flowchart for data

TR 79038/1

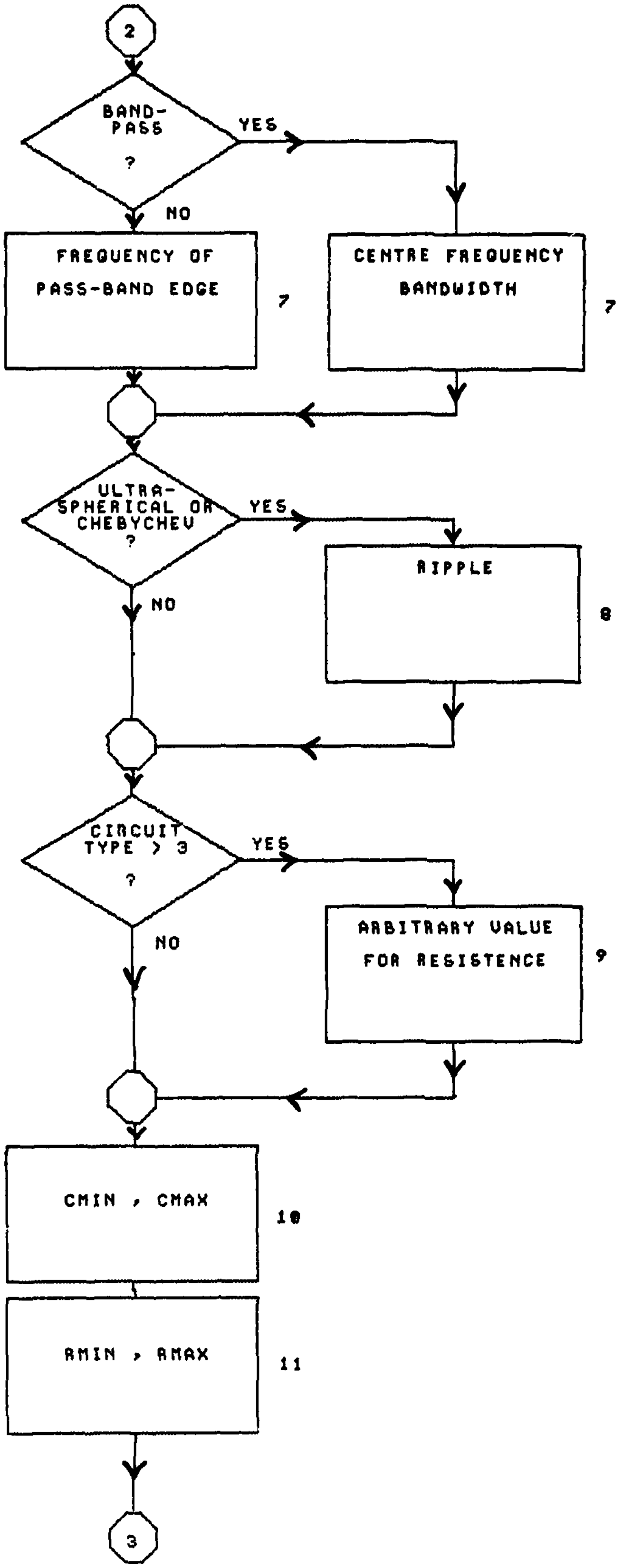

2
BAND-
PASS
?
YES
NO
FREQUENCY OF
PASS-BAND EDGE
7
CENTRE FREQUENCY
BANDWIDTH
7
ULTRA-
SPHERICAL OR
CHEBYCHEV
?
YES
NO
RIPPLE
8
CIRCUIT
TYPE > 3
?
YES
NO
ARBITRARY VALUE
FOR RESISTENCE
9
CMIN , CMAX
10
RMIN , RMAX
11
3

Fig 22 contd

Fig 22 contd

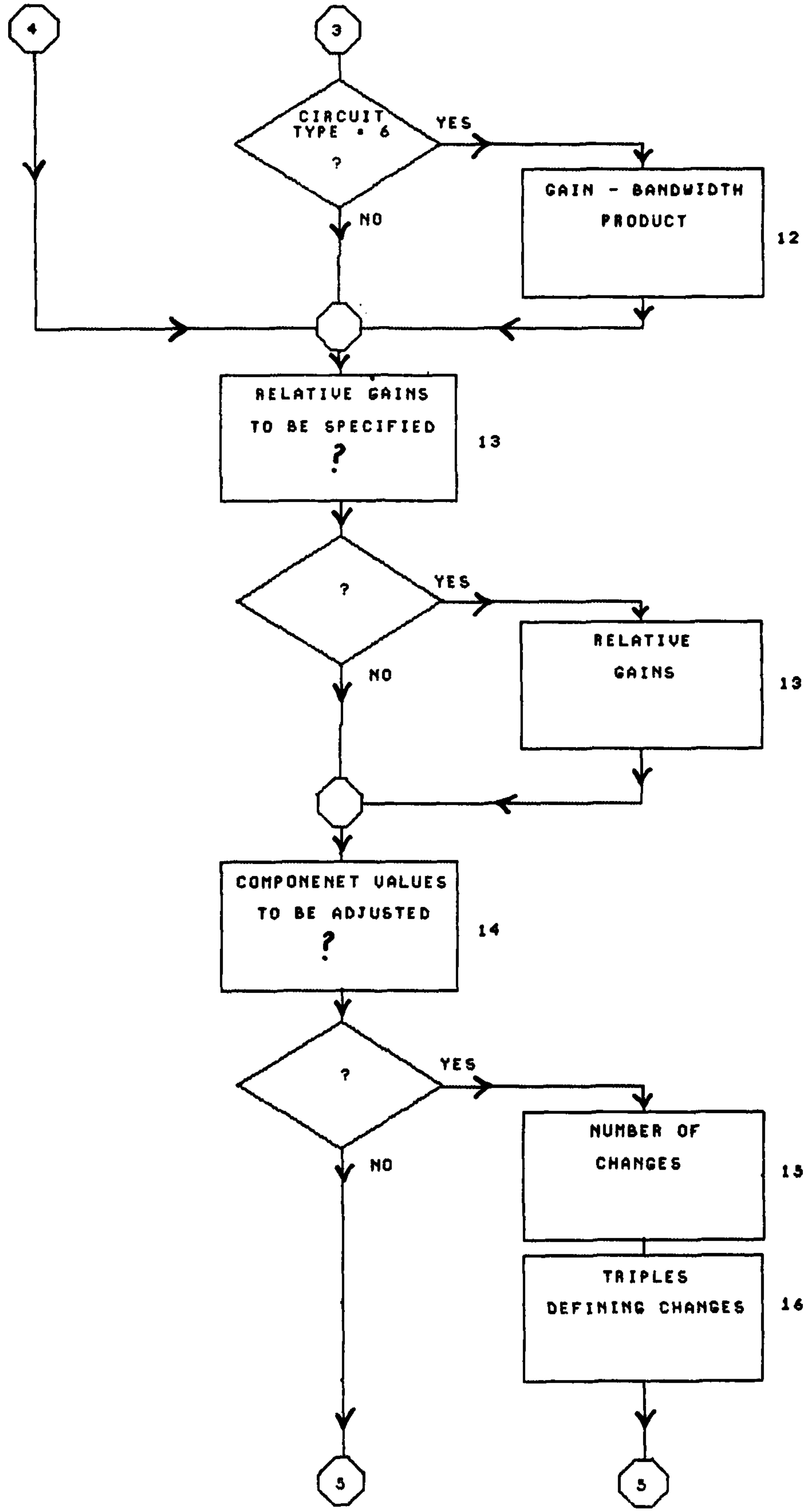


Fig 22 contd

TR 79038/3

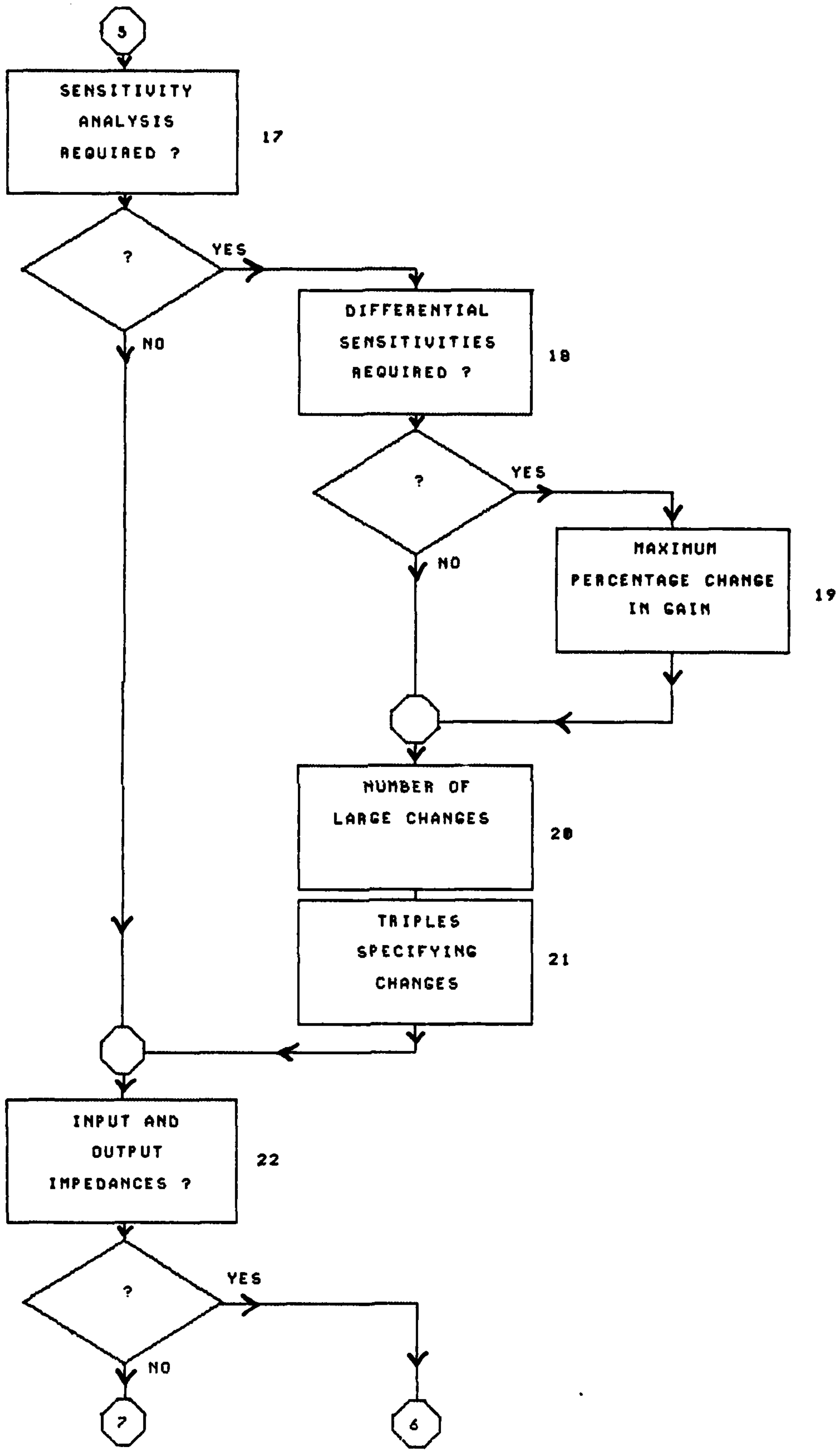

5
SENSITIVITY ANALYSIS REQUIRED ?
17
?
YES
NO
DIFFERENTIAL SENSITIVITIES REQUIRED ?
18
?
YES
NO
MAXIMUM PERCENTAGE CHANGE IN GAIN
19
NUMBER OF LARGE CHANGES
20
TRIPLES SPECIFYING CHANGES
21
INPUT AND OUTPUT IMPEDANCES ?
22
?
YES
NO
7
6

Fig 22 contd

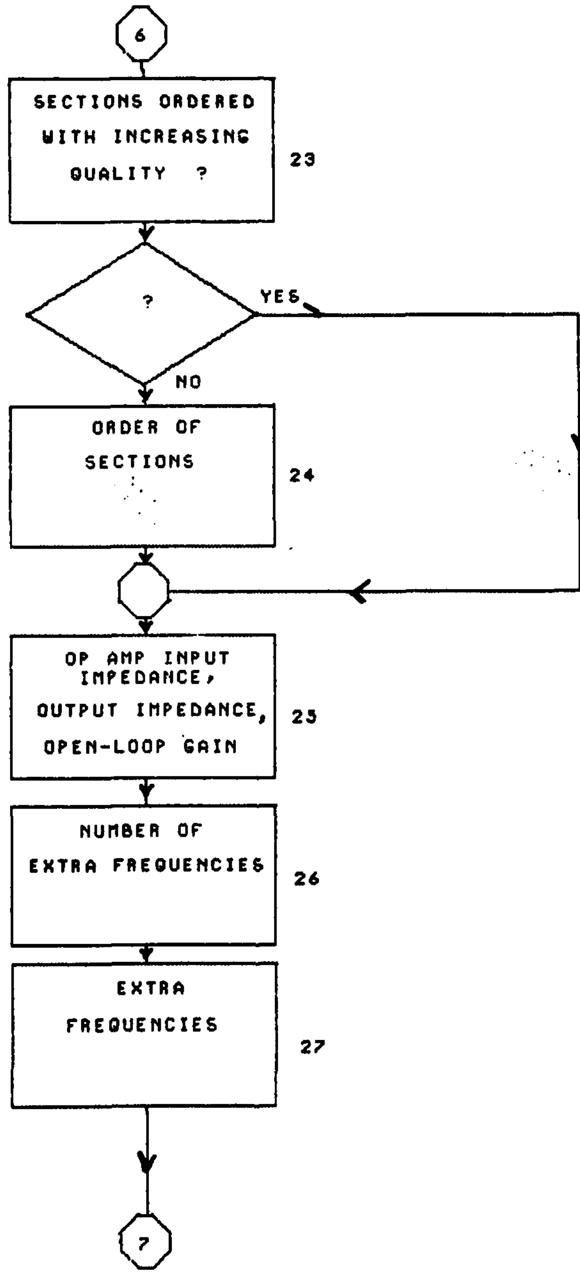

6
SECTIONS ORDERED WITH INCREASING QUALITY ?
23
?
YES
NO
ORDER OF SECTIONS
24
OP AMP INPUT IMPEDANCE, OUTPUT IMPEDANCE, OPEN-LOOP GAIN
25
NUMBER OF EXTRA FREQUENCIES
26
EXTRA FREQUENCIES
27
7

Fig 22 contd

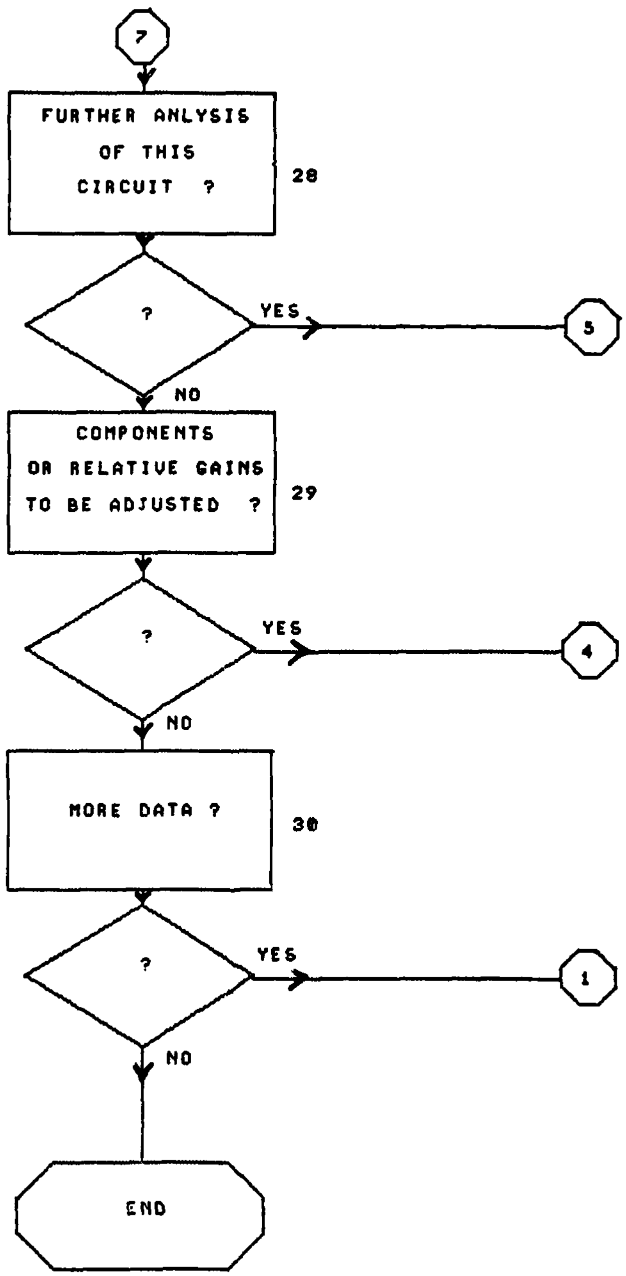


Fig 22 concld

R 79038/6